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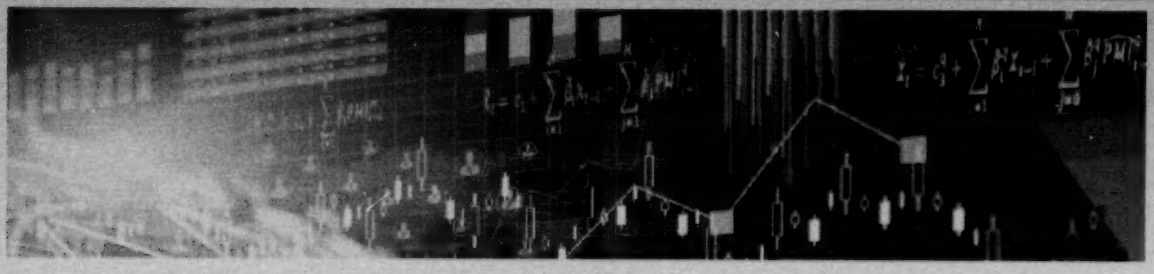
On the Essentiality of E-Money



by Jonathan Chiu and Tsz-Nga Wong

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Abstract

Recent years have witnessed the advances of e-money systems such as Bitcoin, PayPal and various forms of stored-value cards. This paper adopts a mechanism design approach to identify some essential features of different payment systems that implement and improve the constrained optimal resource allocation. We find that, compared to cash, e-money technologies allowing limited participation, limited transferability and non-zero-sum transfers can help mitigate fundamental frictions and enhance social welfare, if they satisfy conditions in terms of parameters such as trade frequency and bargaining powers. An optimally designed e-money system exhibits realistic arrangements including non-linear pricing, cross-subsidization and positive interchange fees even when the technologies incur no costs. Regulations such as a cap on interchange fees (à la the Dodd-Frank Act) can distort the optimal mechanism and reduce welfare.

JEL classification: E, E4, E42, E5, E58, L5, L51
Bank classification: Bank notes; E-money; Payment clearing and settlement systems

Résumé

Ces dernières années ont été marquées par la percée des systèmes de monnaie électronique comme Bitcoin, PayPal et divers types de cartes prépayées. Nous adoptons une démarche axée sur la conception de mécanismes pour repérer certains des éléments essentiels de différents systèmes de paiement qui permettent sous certaines contraintes d'opérer une répartition optimale des ressources et de l'améliorer. D'après nos conclusions, les technologies des systèmes de monnaie électronique qui prévoient des participations limitées, des possibilités de transfert restreintes et des transferts à somme non nulle peuvent, comparativement à l'argent comptant, contribuer à réduire les frictions fondamentales et à accroître le bien-être social, dans la mesure où elles respectent certaines conditions exprimées sous forme de paramètres tels la fréquence des transactions et les pouvoirs de négociation. Un système optimisé de monnaie électronique est assorti de modalités réalistes, parmi lesquelles la tarification non linéaire, l'interfinancement et les frais d'interchange positifs, même si les technologies impliquées n'occasionnent pas de coûts. L'instauration d'une réglementation, comme l'imposition d'un plafond pour les frais d'interchange (sur le modèle de la loi Dodd-Frank), peut altérer le mécanisme optimal et réduire le bien-être.

Classification JEL : E, E4, E42, E5, E58, L5, L51
Classification de la Banque : Billets de banque; Monnaie électronique; Systèmes de compensation et de règlement des paiements

Non-Technical Summary

Recent years have witnessed a number of retail payment innovations known as electronic money (or e-money). While policy-makers are concerned about these new developments, so far there has been limited guidance provided by economic theory regarding the social efficiency of e-money adoption. Building on modern monetary theory, this paper adopts a mechanism design approach to identify essential features of e-money payment systems that help improve efficiency.

We build a micro-founded general equilibrium model to compare the efficiency properties of different payment systems. The starting point is a basic environment in which traditional cash is used as a payment instrument. We then gradually attach to it additional features, including some distinctive characteristics of e-money. Inspired by the latest developments in e-money technologies, we focus on several features of e-money that could be important in improving efficiency but are ignored in policy discussions. First, we consider e-money featuring *limited participation*. The technical possibility of excluding non-compliant traders allows e-money issuers to enforce pre-trade transfers (e.g., membership fees for obtaining e-money devices). Second, we consider an e-money featuring *limited transferability*. The technical possibility of limiting transferability and having non-zero-sum transfers allows the e-money issuer to enforce post-trade charges (e.g., merchant fees and interchange fees).

We show that the above features of e-money are essential because they help achieve cross-subsidization between buyers and sellers and improve efficiency in resource allocation relative to a payment system without these features. Furthermore, we characterize some key properties of an optimally designed e-money mechanism, and provide examples of some simple e-money schemes. Interestingly, an optimally designed e-money system in this simple model exhibits realistic arrangements including non-linear pricing, cross-subsidization and positive interchange fees even when the technologies incur no costs.

The above implications provide useful lessons for policy-makers. First, e-money is fundamentally different from conventional money. Improved information and technologies as a result of the introduction of e-money allow more general fee structures and can increase efficiency. Second, pricing arrangements such as merchant and interchange fees can be essential components of an optimal payment system. Hence, fee regulation may distort the optimal mechanism and reduce welfare. Third, our theory suggests that different payment instruments emerge to mitigate different economic frictions. For example, there is a fundamental difference between money/e-money and credit because consumers need to acquire balances in advance in the former but not in the latter case, implying that the optimal design of a money-based payment system can be quite different from that of a credit-based system. For instance, limiting interchange fees can be optimal for some specific payment instruments, but not all.

1 Introduction

Recent years have witnessed a number of retail payment innovations known as electronic money (or e-money).[1] The latest generation of electronic money substantially improves the performance of payment instruments in terms of convenience, durability and transaction speed.[2] However, although the emergence and adoption of new media of exchange - for example, from Yap stones to shell money to paper money to e-money - have been taking place over the course of history, the basic functioning of the payment system for *monetary* exchange remains largely unchanged. In a monetary payment system, whether a Yap stone or paper money is used, in order to purchase a product, a buyer needs to first acquire the means of payment from others, bring it to the point of sale, and conduct a quid pro quo exchange with the seller, who then uses the means of payment in other transactions. These observations seem to suggest the existence of deep, fundamental frictions that underlie and determine the basic mode of monetary exchange. Payment technologies have certainly evolved over time. It is unclear, though, whether all of these improvements are useful for overcoming the deep frictions that shape the basic functioning of payment systems. The emergence of e-money provides an opportunity for understanding and answering some basic but important questions about payment systems: is e-money merely another kind of sea shells, or instead something fundamentally different from conventional money, something that helps mitigate deep frictions? What are these frictions? How do various payment systems emerge endogenously in response to these frictions? Is there any need

[1] The Survey of Electronic Money Developments by the Committee on Payment and Settlement Systems (CPSS) noted that "in a sizeable number of the countries surveyed, card-based e-money schemes have been launched and are operating relatively successfully: Austria, Belgium, Brazil, Denmark, Finland, Germany, Hong Kong, India, Italy, Lithuania, the Netherlands, Nigeria, Portugal, Singapore, Spain, Sweden and Switzerland. Network-based schemes are operational or are under trial in a few countries (Australia, Austria, Colombia, Italy, the United Kingdom and the United States), but remain limited in their usage, scope and application." (CPSS, 2001)

[2] There is no universal definition for e-money that can fit precisely all exisiting variants of e-money products. One definition of e money proposed by CPSS is the following: it is the "monetary value represented by a claim on the issuers which is stored on an electronic device such as a chip card or a hard drive in personal computers or servers or other devices such as mobile phones and issued upon receipt of funds in an amount not less in value than the monetary value received and accepted as a means of payment by undertakings other than the issuer." This defintion is quite broad (e.g. including debit cards), and at the same time quite narrow (e.g. excluding Bitcoin). Similarly, the European Central Bank defines e-money as "an electronic store of monetary value on a technical device that may be widely used for making payments to entities other than the e-money issuer." For the purpose of this paper, we don't need to stick with one specific definition of e-money. Instead, we will examine several features that are commonly found in e-money products.

for government to regulate e-money payment systems in the presence of these frictions?

To answer these questions, this paper builds on recent developments in monetary theory. It is now widely recognized that in the presence of such deep frictions as the lack of commitment and lack of record-keeping, the use of money as a payment instrument improves the efficiency of resource allocations (Kocherlakota, 1998). In this sense, money, as a medium of exchange, is *essential* because it improves efficiency relative to an economy without money. However, modern monetary theory also teaches us that, in a world subject to frictions that render money essential, the equilibrium allocation is typically suboptimal. This is because the use of money requires pre-investment by impatient buyers, giving rise to a cash-in-advance constraint. In a decentralized economy, this constraint often leads to an inefficient allocation: impatient buyers acquiring too little money, and hence being liquidity constrained in trading (for example, due to discounting and inflation). In addition, a resource misallocation can be magnified by an inefficient trading mechanism whenever the surplus from trade is not allocated in a way respecting the pre-investment of buyers. In Lagos and Wright (2005), all these effects give rise to a so-called "holdup" problem.

The aforementioned frictions that render money essential also shape the basic functioning of monetary payment systems. Owing to its full anonymity and decentralization of trades, the conventional money-based payment system typically exhibits the following features. First, it permits *non-exclusive participation*: anyone can freely participate in the monetary system to hold cash without other prerequisites. Second, it allows *unrestricted transferability*: beyond transaction costs, there is no restriction on the transferability of money balances. Any amount of cash can change hands at any time, anywhere and between any parties. In other words, non-exclusive participation means that there is no limitation on who can use money (the extensive margin), and unrestricted transferability means there is no limitation on how money is used (the intensive margin). In addition, all transfers are *zero sum:* the amount of money balances transferred by the payer is always equal to the amount received by the payee.

We argue that an e-money-based payment system is fundamentally different from the money-based system because it can be free of the above-noted features. First, e-money issuers can exclude certain agents from participation. For example, in card-based e-money schemes such as the Octopus card system in Hong Kong, only buyers

who have already acquired stored-value smart cards and merchants who have obtained card readers/writers from the card operator can participate in the system to conduct payments. Similarly, in server-based e-money schemes such as PayPal, only individual and business users who have already signed up for an account can hold, send and receive e-money balances. In this type of e-money scheme, non-compliance leads to exclusion from the system. Second, e-money systems can restrict balance transfers. For example, in centralized e-money schemes such as PayPal, the system operator maintains user accounts and performs payment processing, and thus has the ability to block or restrict the size or direction of balance transfers.[3] In some decentralized systems such as Bitcoin, bilateral transactions can be completed only after they are verified and written into a general ledger by other users (e.g. Bitcoin miners). In addition, according to CPSS (2001), it is quite common globally that the transferability of e-money balances among end-users is restricted. Specifically, 77% of e-money systems included in that survey prohibit transferability among end-users. Furthermore, since balances are transferred through electronic devices, it is technically feasible to have non-zero-sum transfers: the amount of balances transferred by the payer differs from the amount received by the payee. For example, the payee receives only \$97.1 for every \$100 sent to the payee through the PayPal system. Bitcoin also has a built-in feature that allows the individual making a transaction to include a transaction fee paid to the Bitcoin miner. This feature of e-money can allow for charging merchants fees or other transaction fees, which are often observed in e-money payment systems.[4]

Of course, the fact that e-money is fundamentally different from money does not necessarily mean that it is more essential. Next, we use a mechanism design approach

[3] A decentralized e-money scheme is one in which the payment network is not provided or managed by a single network provider or operator (for example, a system in which e-money is stored and flows through a peer-to-peer computer network that directly links users). In contrast, in a centralized network, there is a trusted third party that manages the payment network. One example is server-based e-money schemes such as PayPal.

[4] Other payment systems (such as credit cards and large-value settlement systems) may also exhibit these features. But these systems usually require monitoring performed by banks, credit card issuers or clearing houses who possess a richer information set and/or a stronger enforcement technology than a typical e-money issuer. It is not clear (i) whether these arrangements (e.g. credit) are feasible in the current environment, and (ii) whether money and e-money remain essential in an environment in which these arrangements (e.g. credit) are feasible. In this regard, the model constructed in this paper may not be a good one for analyzing these systems. In addition, the functioning of many traditional payment systems relies on accounts with identified owners, making it inaccessible for some populations or unavailable in some situations. In contrast, electronic money such as prepaid cards and Bitcoins can function even in a setting with anonymous users – a setting that renders cash essential.

to analyze whether any of e-money's distinctive features also make it more essential: e-money is more essential than money if the use of e-money allows the implementation of some socially desirable allocations that are not implementable with the use of money.[5] We build a micro-founded general equilibrium model to capture these e-money features and compare the efficiency properties of different payment systems. The starting point is a basic environment in which traditional cash is used as a payment instrument. We then gradually attach to it additional features, including some distinctive characteristics of e-money. We identify several features of e-money that can help mitigate fundamental frictions and enhance efficiency in a cash economy. First, we consider e-money featuring *limited participation*. The technical possibility of excluding non-compliant traders allows e-money issuers to enforce pre-trade transfers (e.g. membership fees for obtaining e-money devices). Second, we consider an e-money featuring *limited transferability*. The technical possibility of limiting transferability and having non-zero-sum transfers allows the e-money issuer to enforce post-trade charges (e.g. merchant fees and interchange fees).[6]

We show that only under some conditions will the introduction of e-money relax certain binding constraints faced by the money issuer and allow more flexible and efficient intervention. As a benchmark, the first main finding of our paper is that an inefficient allocation can arise even in an *optimally designed* monetary system (subject to non-exclusive participation, unrestricted transferability and zero-sum transfers). The second main finding of our paper is that certain new features of e-money are essential because they help achieve cross-subsidization between buyers and sellers and improve efficiency in resource allocation relative to a payment system without these features. Interestingly, we show that e-money with limited transferability can be more or less able to achieve efficiency than one with limited participation, depending on such primitives as buyers' bargaining power and the frequency of trade. Finally, we characterize some key properties of an optimally designed e-money mechanism, and provide examples of simple direct and indirect mechanisms.

[5] See Wallace (2010) for an introduction of the mechanism design approach to monetary economics.

[6] While some e-money systems allow the issuer to track the identity and payment history of users, it can be difficult to implement in most anonymous systems (e.g., Bitcoin and prepaid card). One future extension of this paper is to explore the welfare implication of introducing a record-keeping technology into this environment. One would expect that giving an additional technology to the e-money mechanism designer should only make it easier to achieve desirable allocations.

Finally, our paper is highly relevant to recent policy discussion. Developments in payment technologies raise new challenges for policy-makers. The Federal Reserve System, for example, has been soliciting public inputs on strategies and tactics for reforming the U.S. payment system.[7] More specifically, in the "Survey of Electronic Money Developments," the Bank for International Settlements highlighted that "Electronic money projected to take over from physical cash for most if not all small-value payments continues to evoke considerable interest both among the public and the various authorities concerned, including central banks." (CPSS, 2001) Against this context, the Bank of Canada has developed an active research agenda to understand and monitor e-money products.[8] While policy-makers are definitely concerned about these new developments, so far there has been limited guidance provided by economic theory regarding the welfare implications of e-money adoption. To the best of our knowledge, no existing research on e-money performs welfare analysis giving serious consideration to fundamental frictions in payment systems. While modern monetary theory focuses on understanding the fundamental roles of conventional money and credit, the role of e-money has not yet been explored. Our paper is also the first to develop a micro-founded general equilibrium model of e-money. By uncovering essential features of a payment instrument such as e-money, our results can provide guidance to policy-makers on how to design the future payment systems, as well as whether and how an e-money system should be regulated. For example, our model can be used to evaluate the effect of imposing a cap on interchange fees in an e-money system (similar to that introduced by the Durbin Amendment to the Dodd-Frank Act[9]).

Literatures

Our paper is directly related to the literature of monetary theory. In general, this literature focuses on an economic environment in which contracts involving intertemporal obligations are infeasible, due to frictions such as the lack of commitment and lack of record-keeping, and in which money is the only durable object that can serve as a means of payment. Lagos and Wright (2005) develop a tractable framework with the

[7]See the Payment System Improvement - Public Consultation Paper (https://fedpaymentsimprovement.org/wp-content/uploads/2013/09/Payment_System_Improvement-Public_Consultation_Paper.pdf).

[8]See, for example, the webpage http://www.bankofcanada.ca/research/e-money.

[9]See, for example, Wang (2012) for details and discussion of this interchange fee regulation.

presence of these frictions for studying the roles of money and monetary policy. Recent models of payment systems building on the Lagos-Wright framework include Kahn and Roberds (2009), Li (2011) and Monnet and Roberds (2008).[10] Within this literature there is a line of research using the mechanism design approach to study the essentiality of money and other means of payment.[11] To the best of our knowledge, this paper is the first in this literature to study the essentiality of e-money. This question is non-trivial because new payment technologies such as e-money do not necessarily outperform conventional money. We show how the essentiality of e-money depends on primitives such as preferences, technology, and agents' bargaining power, and characterize the optimal arrangements of an e-money payment system.

As mentioned above, in a monetary economy, the socially optimal allocation (the first best) typically cannot be implemented without an appropriately designed mechanism. Moreover, the implementation of the constrained optimal allocation (the second-best) is usually not unique. There are two strands of research, both taking the payment system as given, but which focus on different implementation mechanisms. The first strand takes an inefficient trading protocol as a primitive, and studies the design of monetary policy to mitigate this inefficiency.[12] For example, Lagos and Wright (2005) and Lagos (2010) find that the Friedman rule is optimal in these environments, but it involves taxing agents, which is not first best because agents will not voluntarily pay taxes. With the use of a fixed fee and linear transfers, Andolfatto (2010) illustrates how the first best can be implemented with voluntary participation in a competitive environment.

The second strand of research, including Hu, Kennan, and Wallace (2009) and Ro-

[10] Another line of research is the two-sided market literature in the field of industrial organization. See the surveys by Rochet and Tirole (2003), Kahn and Roberds (2009) and Rysman (2009). This literature typically studies a partial equilibrium setting and assumes a particular form of fee structures. See Shy and Wang (2011) for a recent study on interchange fees, which uses the two-sided market approach to analyze a payment environment related to ours.

[11] Kocherlakota (1998) is the first to use implementation theory to show the essentiality of money in a random matching environment. Araujo, Camargo, Minetti and Puzzello (2012) show the essentiality of money in the Lagos-Wright environment that is used in this paper. See Kocherlakota and Wallace (1998) on the essentiality of money and credit; Kocherlakota (2003) on illiquid bonds; Hu and Rocheteau (2013) on money and high-yield assets; Hu and Zhang (2014) on money and capital with endogenous search intensity.

[12] This literature is growing. See Araujo and Hu (2014) on the optimal quantitative easing in an economy with money and credit; Cavalcanti and Erosa (2008) on the propagation of shocks in monetary economies; Cavalcanti and Nosal (2009) on cyclical monetary policy; Wallace (2014) on the optimal monetary transfer with non-degenerate distribution of money.

cheteau (2012), takes the suboptimal monetary policy as given, and designs the trading protocol to mitigate the resulting inefficiency. These studies endogenize the trading protocol using a mechanism design approach, as advocated by Wallace (2010). This literature finds that, under certain conditions, the first best can still be implemented by adopting an optimal trading protocol in pairwise trades. Specifically, deviation from Friedman's rule can still be optimal, and the welfare cost of inflation can be zero.

Our paper is related to both strands of research. Unlike the first strand such as Lagos and Wright (2005) and Andolfatto (2010), we do not restrict ourselves to any particular type of intervention, and use the mechanism design approach to endogenize the payment instruments and payment system. Another key difference from Andolfatto (2010) is that we model matching frictions and inefficient bargaining, rather than centralized trading, which tends to understate the distortions and hence overstate the power of policy, as argued in Hu, Kennan, and Wallace (2009). It makes our economy a robust setting for optimal policies. Unlike the second strand such as Hu, Kennan, and Wallace (2009) and Rocheteau (2012), we take inefficient trading protocols as one of the primitive inputs to the mechanism design of the payment system. Our perspective is particularly relevant for policy-makers, such as central banks and payment system regulators, who arguably have limited influence over the determination of terms-of-trade in a decentralized and anonymous situation. Overall, the mechanism design approach is powerful, since it can help identify the essential features of e-money and clarify their role in the payment system.

The rest of the paper is organized as follows. Section 2 presents the model environment. Section 3 designs the optimal money mechanism, highlighting the importance of a non-linear scheme and its limitation. Section 4 designs the optimal e-money mechanism with limited participation, highlighting the importance of cross-subsidization and its limitation. Section 5 designs the optimal e-money mechanism with limited transferability, highlighting the importance of after-trade fees and their essentiality. Section 6 extends the analysis to consider competitive pricing. Section 7 concludes.

2 Baseline Model

Our model is based on the alternating market formulation from Rocheteau and Wright (2005). The economy is populated with two types of agents: measure one of buyers and

measure one of sellers. Time is discrete and infinite, indexed by $t = 0, 1...$ Alternating in each period are subperiods of day and night: during the day a frictional decentralized market (DM) convenes where buyers and sellers match randomly and bilaterally; during the night a frictionless centralized market (CM) convenes where agents trade with each other at Walrasian prices. Goods traded in the CM and DM are denoted, respectively, as CM goods and DM goods. In the DM, agents can only observe the actions and outcomes of their trades, and are anonymous. There is no technology for monitoring, enforcement or coordinating global punishment. As a result, credit is infeasible and a medium of exchange - money - is essential for trades in the DM. The stock of money is denoted by M_t, which has an exogenous growth rate μ so that $M_{t+1} = \mu M_t$. Let ϕ_t be the price of money in terms of the (numeraire) CM goods. New money is introduced by lump-sum transfers such that each agent receives $T_t = (\mu - 1)\,\phi_t M_t/2$ transfer of real balances in the CM. As seen later, the real balances $z \equiv \phi m$ are the relevant state variable for an agent's decisions.

Technology and preference

Agents live forever with a discount factor $\beta \in (0, 1)$. Utility in a period depends on actions in the CM and DM. In the CM, all agents can consume/produce the numeraires (with $l > 0$ denoting consumption and $l < 0$ production) and have linear preferences over l. In the DM, buyers can consume the DM goods, denoted by q, produced by sellers. The utilities of buyers and of sellers are given by

$$\begin{aligned} \widetilde{U}_b(q, l) &= U(q) + l, \\ \widetilde{U}_s(q, l) &= -C(q) + l, \end{aligned}$$

where $U(q)$ is the buyer's utility function and $C(q)$ is the seller's cost function in the DM. We assume that $U' > 0$, $U'' < 0$, $U(0) = 0$ and $\lim_{q \to 0} U'(q) = \infty$; $C(0) = 0, C'(q) \geq 0, C''(q) \geq 0$ and $C'(0) = 0$. An agent's lifetime preferences are given by $\mathbb{E} \sum \beta^t \widetilde{U}_j(q_t, l_t)$, $j = b, s$.

The agent's problem is as follows. We denote the value functions of a type $j = b, s$ with real balances $\widetilde{z}$ in the CM and z in the DM by $W_j(\widetilde{z})$ and $V_j(z)$, respectively. In

the CM, the agent's budget constraint is

$$\frac{\phi}{\phi_{+1}} z + l = \tilde{z} + T, \tag{1}$$

where ϕ/ϕ_{+1} is the inflation factor capturing the change of the real price of money across periods (time subscript t is omitted). An agent chooses real balances z to be brought into next period's DM, which is financed by initial CM real balances $\tilde{z}$, sales $-l$ of the numeraire and the transfer T from the central bank. Due to the quasi-linear utility and the CM budget (1), the CM problem is given by

$$W_j(\tilde{z}) = \max_z \left\{ \tilde{z} + T - \frac{\phi}{\phi_{+1}} z + \beta V_j(z) \right\}. \tag{2}$$

Decision in DM

Next we turn to the DM problem. In the DM, buyers and sellers are subject to pairwise random matching. With probability α, a buyer (seller) is matched with a seller (buyer), and with probability $1-\alpha$, there is no match. Consider a buyer with real balances z_b matching a seller with z_s in the DM. The trade surpluses of the buyer and of the seller in a DM match are defined, respectively, as

$$\begin{aligned} S_b(q,d;z_b,z_s) &\equiv U(q) + W_b(z_b - d) - W_b(z_b), \\ S_s(q,d;z_b,z_s) &\equiv -C(q) + W_s(z_s + d) - W_s(z_s), \end{aligned}$$

where d is the payment of real balances by the buyer. The terms of trade (q,d) solve the following proportional bargaining problem[13]:

$$\max_{q,d} \{S_b(q,d;z_b,z_s) + S_s(q,d;z_b,z_s)\}, \tag{3}$$

subject to the bargaining rule:

$$S_b(q,d;z_b,z_s) = \theta[S_b(q,d;z_b,z_s) + S_s(q,d;z_b,z_s)],$$

where $\theta \in (0,1]$ is the buyer's bargaining power, as well as the liquidity constraint

$$d \in [-z_s, z_b].$$

[13] We consider below competitive pricing and show that the main result is not affected.

Since $W_b(\tilde{z})$ and $W_s(\tilde{z})$ are linear in $\tilde{z}$, the bargaining solution (q,d) depends only on the buyer's money balance z_b, denoted as $\{q(z_b), d(z_b)\}$. Define q^* as the first-best CM consumption satisfying $U'(q^*) = C'(q^*)$. It is straightforward to show that the bargaining solution $\{q(z), d(z)\}$ is given by the following lemma.

Lemma 1 *The bargaining solution $\{q(z), d(z)\}$ satisfies*

$$\begin{aligned} d &= \min\{z, D(q^*)\}, \\ q &= D^{-1}(d), \end{aligned}$$

where

$$D(q) \equiv (1-\theta)U(q) + \theta C(q).$$

Proof. Omitted. ■

Intuitively, when the buyer brings enough money balances to finance the first-best consumption (i.e. $z \geq D(q^*)$), then unconstrained trade is conducted with terms of trade given by q^* and $D(q^*)$. However, the buyer who is constrained (i.e. $z < D(q^*)$) spends all $d = z$, to buy $q = D^{-1}(z) < q^*$. Before matching, the value function of an agent with z in the DM, $V_j(z)$, $j = b, s$, is given by

$$V_b(z) = \alpha[U[q(z)] + W_b[z - d(z)]] + (1-\alpha)W_b(z), \tag{4}$$

$$V_s(z) = \alpha \int [-C[q(z_b)] + W_s[z + d(z_b)]]\, dF(z_b) + (1-\alpha)W_s(z), \tag{5}$$

where F is the cumulative distribution of the buyer's real balances.

Equilibrium

Define a stationary degenerate monetary equilibrium as follows:

Definition 1 *A stationary degenerate monetary equilibrium consists of the price system $\{\phi_t\}_{t=0}^{\infty}$, the allocation (q, z_b, z_s) and the policy $\{M_t, \mu, T\}$, such that*

a. (agent's optimization) given $z_{b,0}$ and $\{\phi_t\}_{t=0}^{\infty}$, z_b and z_s solves (2);

b. (money markets clear) $\phi_t M_t = z_b + z_s$;

c. (bargaining) $q = q(z_b)$ solves (3) ;

d. (issuer's budget constraint) given ϕ_t, $\{M_t, \mu, T\}$ satisfies $T = (\mu - 1)\phi_t M_t$;

e. (monetary, stationary) $\phi_t > 0, \phi_t/\phi_{t+1} = \mu$.

Since $W_s(\tilde{z})$ and $V_s(z)$ are linear, if a seller buys money in the period t CM and resells it in the period $t+1$ CM, the rate of return in terms of utility is $\beta\phi_{t+1}/\phi_t - 1 = \beta/\mu - 1$. Therefore, whenever $\mu < \beta$, the seller's money demand z_s is infinite, and hence the money market cannot be cleared. On the other hand, when $\mu > \beta$, we must have $z_s = 0$. Intuitively, sellers have no need to spend money in the DM, and thus they have no incentive to buy money in the CM as long as its rate of return is negative (i.e. $\mu > \beta$). Similarly, a buyer will choose to bring an infinite amount for the next CM when $\mu < \beta$. When $\mu > \beta$, a buyer will not bring a money balance that is not intended to be used in a DM match. In other words, the cash-in-advance constraint, $d \leq z_b$, is always binding in the DM when $\mu > \beta$. In this case, using Lemma 1 and ignoring the constant terms, we can rewrite the buyer's optimization problem (2) in the CM as

$$\max_q \{-[\mu - \beta(1-\alpha)]D(q) + \beta\alpha U(q)\}, \text{ s.t. } q \leq q^*. \tag{6}$$

Intuitively, a buyer chooses q in the DM and acquires the real balance for the DM trade $D(q)$. With probability α, there is a trade and the benefit from consumption is $\beta U(q)$. With probability $1-\alpha$, the money holding is not spent and has a continuation value of $\beta D(q)$. Therefore, $[\mu - \beta(1-\alpha)]D(q)$ captures the net cost of acquiring the money. A buyer will choose $q = z_b = 0$ when $\beta\alpha U'(0) - [\mu - \beta(1-\alpha)]D'(0) < 0$, simplified to

$$z_b = 0 \Leftrightarrow \mu \geq \bar{\mu} = \beta\left(1 + \frac{\alpha\theta}{1-\theta}\right).$$

Therefore, the monetary equilibrium does not exist when $\mu \geq \bar{\mu}$. For a buyer, since the opportunity cost of carrying nominal balances is increasing in the money growth rate μ, and the return from carrying balances for trade is also increasing in the bargaining weight θ, there is no incentive to hold money when μ is too high, α is too low or θ is too low. These are the three inefficiencies highlighted in the monetary literature: the cash-in-advance constraint, the search friction and the holdup problem. The following proposition characterizes the equilibrium.

Proposition 1 *A monetary equilibrium exists iff $\mu \in [\beta, \bar{\mu})$. If $\mu > \beta$, then $q < q^*$; if $\mu \to \beta$, then $q = q^*$.*

According to this proposition, the first-best allocation with $q = q^*$ cannot be supported when $\mu > \beta$. The idea is that, to consume q^* in the next DM, a buyer

needs to acquire $\mu D(q^*)$ money balances in the CM. So the marginal utility gain with respect to q is $\beta\alpha U'(q^*)$, while the marginal cost of acquiring the balance is $[\mu - \beta(1-\alpha)]D'(q^*) = [\mu - \beta(1-\alpha)]U'(q^*)$. As a result, a buyer has an incentive to marginally reduce q below q^* when

$$(\beta - \mu)U'(q^*) < 0, \tag{7}$$

which is true whenever $\beta < \mu$.

So deflating the economy at the discount rate is necessary and sufficient for implementing the first-best allocation. Furthermore, the money issuer's budget constraint implies that a positive lump-sum *tax* is needed (i.e., a negative transfer $T_t = (\beta - 1)\phi_t M_t < 0$) to implement the first best. If the money issuer has no taxation power, then this simple lump-sum transfer scheme cannot implement the first best. The natural question is: can the first best be supported by using more general transfer schemes (than a lump-sum scheme)? The question calls for a mechanism design approach to examine general transfer functions.

Summary

In this section, we learned that in a monetary economy with lump-sum transfers,

1. a monetary equilibrium does not exist when money growth is too high, trades are too infrequent or the buyer's bargaining power is too low;
2. without taxation, the first-best allocation can never be achieved.

3 Optimal Money Mechanism

The previous section showed that the first-best allocation is not implementable by a basic, lump-sum transfer scheme. However, the previous setting understates the power of mechanism design. A natural question is whether the first-best allocation is achievable when the money issuer can implement *any* (including sophisticated) non-lump-sum transfer scheme. In this regard, we use a mechanism design approach to design the optimal mechanism and interpret the money issuer as the mechanism designer who can conduct an intervention at night *after* the CM is closed. We consider the following

information structure: the money issuer can distinguish between buyers and sellers, but cannot observe an agent's past actions, nor the money balances brought from the CM - the lack of record-keeping that still renders money essential for the DM trades. The relevant space of agent types is thus two-dimensional: whether the agent is a buyer or a seller (observable), and how much money the agent holds (unobservable).

Thanks to the revelation principle, any equilibrium allocation of a Bayesian game under a mechanism can be implemented by a direct mechanism, where agents report their private information to the mechanism designer (here reporting the money balances to the money issuer), and the mechanism designer makes monetary transfers based on the report. For notational convenience, we will assume that the report is about the post-transfer balance z and the pre-transfer balance can be inferred directly. An agent who leaves the CM market with z and decides to skip the mechanism will end this subperiod with exactly z. But an agent who plans to participate in the mechanism will need to report the balance to the issuer. Notice that while it is feasible for an agent to under-report any amount $\widehat{z} \leq z$ (i.e. hiding money), we assume that it is infeasible to over-report any amount $\widehat{z} > z$, since any over-reporting can be verified (also known as the show-me-the-money constraint). Given the report, the money issuer will transfer the agent $T_j(\widehat{z})$ (to charge if negative), which in general can be any function of the agent's type $j = s, b$ and of the report $\widehat{z}$. Formally, a money mechanism $\mathcal{M} = \{T_b(\widehat{z}), T_s(\widehat{z}), \mu\}$ consists of transfer functions for buyers, $T_b(\widehat{z})$, and for sellers, $T_s(\widehat{z})$, and an inflation factor μ.

CM and DM decision

A type $j = b, s$ agent's DM value function under a money mechanism $\mathcal{M}$ remains the same, given by (4) and (5). A type j's CM value function becomes

$$W_j(\widetilde{z}) = \widetilde{z} + \max_{z, \widehat{z}, e_j \in \{0,1\}} \{-\mu z + e_j T_j(\widehat{z}) + \beta V_j(z)\}, \text{ s.t. } \widehat{z} \leq z, \tag{8}$$

where $e_j = 1$ and $e_j = 0$ denote, respectively, the decision as to whether to participate or not in the mechanism. Here, an agent with z chooses to report $\widehat{z}$ subject to the constraint that over-reporting is not feasible, and the agent's report will result in payment $T_j(\widehat{z})$.

Incentive-compatibility for buyers

It is straightforward to establish that the bargaining solution under a money mechanism $\mathcal{M}$ is still characterized by Lemma 1. Using the linearity of $W_b(\tilde{z})$ and ignoring the constant terms, we can reformulate the buyer's problem under mechanism $\mathcal{M}$ as

$$\max_{q,\hat{z},e_b\in\{0,1\}} \{e_b T_b(\hat{z}) - [\mu - \beta(1-\alpha)] D(q) + \beta\alpha U(q)\}, \text{ s.t.} \tag{9}$$

$$\hat{z} \leq D(q). \tag{10}$$

Here, a buyer who decides not to participate in the mechanism just brings $z = D(q)$ to the next DM, which will allow the buyer to consume q. A buyer who decides to participate and intends to have a post-transfer balance of $z = D(q)$ needs to bring $\mu D(q) - T_b(\hat{z})$ from the CM, and report $\hat{z} \leq D(q)$, which will allow the buyer to consume q in the following DM.

Definition 2 *An allocation (q, z_b, z_s) is incentive compatible for buyers under a money mechanism $\mathcal{M}$ if $e_b = 1, \hat{z} = z_b$, and $q = q(z_b)$ solves (9).*

To induce buyers to participate in the mechanism (i.e. $e = 1$), it is necessary to have an incentive-compatible allocation (q, z_b, z_s) satisfying

$$T_b(z_b) - [\mu - \beta(1-\alpha)] D(q) + \beta\alpha U(q) \geq \max_{q'} \{-[\mu - \beta(1-\alpha)] D(q') + \beta\alpha U(q')\}. \tag{11}$$

Here, the left-hand side (LHS) captures the payoff for participating in the mechanism, and the right-hand side (RHS) captures the payoff for skipping it.

Incentive-compatibility for sellers

Similarly, using the linearity of $W_s(\tilde{z})$, and ignoring the constant terms, one can reformulate the seller's problem in the CM as

$$\max_{z,\hat{z},e_s\in\{0,1\}} \{-\mu z + e_s T_s(\hat{z}) + \beta z\}, \text{ s.t. } \hat{z} \leq z. \tag{12}$$

Here, a seller not participating in the mechanism has no reason to bring money and thus the additional payoff is zero. A seller who decides to participate and intends to have a post-transfer balance of z needs to bring $\mu z - T_s(\hat{z})$ from the CM, and report $\hat{z} \leq z$, and this balance will have a continuation value βz.

Definition 3 *An allocation (q, z_b, z_s) is incentive compatible for sellers under a money mechanism $\mathcal{M}$ if $e_s = 1$ and $\widehat{z} = z = z_s$ solve (12).*

Notice that, conditional on $e_s = 0$, the value of (12) is zero. So to induce sellers to participate in the mechanism, it is necessary to have an incentive-compatible allocation (q, z_b, z_s) that satisfies

$$(\beta - \mu) z_s + T_s(z_s) \geq 0. \tag{13}$$

Here, the LHS captures the payoff for participating in the mechanism, and the RHS captures the payoff for skipping it.

Money issuer's budget constraint

A money issuer has to balance its budget, or self-finance:

Definition 4 *A money mechanism $\mathcal{M} = \{T_b(z), T_s(z), \mu\}$ is self-financed under the allocation (q, z_b, z_s) if*

$$T_b(z_b) + T_s(z_s) = (\mu - 1)(z_b + z_s). \tag{14}$$

This budget constraint states that the issuer's total expenditure on transfers (LHS) has to be financed by money creation (RHS). The total real balances are $z_b + z_s$ at the beginning of a period, and $\mu(z_b + z_s)$ at the end of a period. The RHS denotes the total balances created within a period. Notice that if an allocation (q, z_b, z_s) is incentive compatible for buyers and sellers under a mechanism $\mathcal{M}$, then the equilibrium conditions in the definition are satisfied.

Implementation of the first best

We are interested in whether the first-best allocation can be implemented. The following definition introduces this concept formally.

Definition 5 *A money mechanism $\mathcal{M}$ implements the first best if*

a. there exists (z_b, z_s) such that the first-best allocation (q^*, z_b, z_s) is incentive compatible for buyers and sellers; and

b. $\mathcal{M}$ is self-financed under the first-best allocation (q^*, z_b, z_s).

Define a threshold level $\overline{\theta}$ of the buyer's bargaining power given by

$$\overline{\theta} \equiv \frac{1-\beta}{[1-\beta(1-\alpha)]\left[1-\frac{C(q^*)}{U(q^*)}\right]},$$

which is decreasing in β and α. Note that $\overline{\theta} \in (0,1)$ iff

$$\frac{\beta\alpha}{1-\beta(1-\alpha)}U(q^*) > C(q^*). \tag{15}$$

We are going to assume that this condition holds throughout the paper. Intuitively, when a buyer brings cash from the CM to finance the first-best trade q^* with the first seller matched in the subsequent DMs, the maximum (discounted) utility gain from this future DM trade is $\beta\alpha U(q^*)[1+\beta(1-\alpha)+\beta^2(1-\alpha)^2+...] = \beta\alpha U(q^*)/[1-\beta(1-\alpha)]$. The minimum price the buyer needs to pay to induce the seller to trade is $C(q^*)$. When the above condition is violated, the maximum gain is lower than the minimum price, and hence there is no hope for first-best trade in a monetary economy in which agents need to bring cash to trade.

The following proposition characterizes the implementability of the first-best allocation under an optimally designed money mechanism.

Proposition 2 *There exists a money mechanism $\mathcal{M}$ that implements the first best if and only if $\theta \geq \overline{\theta}$.*

Proof. See the appendix. ■

In the previous section, Proposition 1 shows that, without any authority to enforce taxation, the first-best allocation cannot be achieved by simple lump-sum transfers. This is because buyers have an incentive to marginally reduce q below q^* when (7) is satisfied (i.e. $\beta < \mu$). According to Proposition 2, a well-designed mechanism can still implement the first best. To do that, the transfer scheme $T_b(z)$ has to be designed to induce buyers to carry the right amount of money balances to finance the first-best trade. In particular, when the transfer scheme is non-linear and optimally designed, a buyer no longer has the marginal incentive to reduce q below q^* even when $\beta < \mu$. To give a concrete example, a mechanism may make a big transfer to buyers who bring and report a sufficiently high money balance, and make no transfers to buyers who bring and report too little balances. In an equilibrium in which all buyers co-operate and

receive big transfers, inflation is high. Hence, a deviator who brings too little money and receives no transfers will suffer a loss in purchasing power. Under this non-linear scheme, buyers do not want to lower their money holding too much because that will significantly reduce their surplus from DM trades. This explains why the first best can be supported by the optimal mechanism. However, the power of this scheme is limited by the size of a buyer's DM trade surplus, which in turn depends on θ. That explains why the first best can no longer be supported when θ is too low.

Characterization of optimal mechanism

The above discussion suggests that, to support the first best, transfers to buyers are needed, and hence money growth is positive. The following proposition formally establishes this finding, which characterizes all optimal money mechanisms.

Proposition 3 *If a money mechanism* $\mathcal{M} = \{T_b(z), T_s(z), \mu\}$ *implements the first best, then* $\mu > 1$.

Proof. Suppose there exists a mechanism $\mathcal{M} = \{T_b(z), T_s(z), \mu\}$ that implements the first best with $\mu \leq 1$. Then from the proof of Proposition 2, we have

$$\begin{aligned} -[1-\beta(1-\alpha)]D(q^*) + \beta\alpha U(q^*) &\geq T_b(z_b) - [\mu - \beta(1-\alpha)]d^* + \beta\alpha U(q^*), \\ &\geq \max_{q'}\{-[\mu-\beta(1-\alpha)]D(q') + \beta\alpha U(q')\}, \\ &\geq \max_{q'}\{-[1-\beta(1-\alpha)]D(q') + \beta\alpha U(q')\}, \end{aligned}$$

which is a contradiction, since q^* is the maximizer to the last line only if $\beta = 1$. ■

Simple examples

To illustrate the basic idea, suppose that $\alpha = 1$. We propose a simple example of a direct mechanism, and then an example of an indirect mechanism.

(i) Direct mechanism

This simple mechanism makes no transfers to sellers, so $T_s(z) = 0$ for all z. The money growth is set to $\mu = \overline{\mu}$ and the money created is used to finance transfers to buyers such that

$$T_b(z) = \begin{cases} (\overline{\mu} - 1)D(q^*), \text{ if } z = D(q^*) \\ 0, \text{ otherwise.} \end{cases}.$$

First, note that $\mu > 1$ when $\theta \geq \overline{\theta}$, so that a buyer with $z = D(q^*)$ can receive a positive transfer $(\overline{\mu} - 1) D(q^*) > 0$ from the money issuer. As a result, this buyer has a payoff which equals to $-D(q^*) + \beta U(q^*)$, which is positive iff $\theta \geq \overline{\theta}$. Under this mechanism, a buyer with $z \neq D(q^*)$ does not receive transfers, and thus has a non-positive payoff because $\mu = \overline{\mu}$. Notice that this scheme is non-linear with respect to the buyer's money holding z.

(ii) Indirect mechanism: fixed fee and interest payments

Following Andolfatto (2010), we derive in Appendix A an indirect mechanism with the following features: the money issuer imposes a fixed fee B on buyers, who can then collect interest on their money balances at the rate R at the end of the CM:

$$\begin{aligned} R &= \frac{\mu}{\beta} - 1, \\ B &= (1 - \beta) D(q^*). \end{aligned}$$

We show that, for sufficiently high μ, the first-best allocation can be supported if $\theta \geq \overline{\theta}$. The basic idea is that the interest payment offsets the buyers' opportunity cost of carrying money balances to the DM. This interest payment is financed by the fixed fee paid by the buyers. In order to induce them to pay this fee, the monetary growth has to be sufficiently high so that non-participants' trade surplus in the DM is sufficiently low. Notice that this scheme is piecewise linear: a fixed fee plus a linear transfer with respect to the buyer's money holding z.

Summary

In this section, we learned that:

1. the first best cannot be achieved by any money mechanism when the buyer's bargaining power is too low, the buyer is too impatient or trades are too infrequent;
2. a non-linear transfer scheme and monetary expansion are essential features of a money mechanism to implement the first best;
3. the first best can be implemented by a simple indirect mechanism with interest on buyers' balances, financed by fixed fees and monetary expansion.

4 Electronic Money with Limited Participation

The type of money modelled so far is rather primitive – merely a durable, inert object for circulation. To capture recent innovations, we introduce different payment technologies. Suppose there is an e-money issuer who maintains the supply of another medium of exchange, *e-money*, in addition to money. E-money shares all the basic properties of money: it is divisible, durable, portable and cannot be counterfeited. However, as noted in the introduction and will be discussed in greater detail in this section, e-money is fundamentally different from money. As with the money issuer in the previous section, the e-money issuer can distinguish between buyers and sellers. But the e-money issuer does not have any record-keeping technology, and therefore cannot observe an agent's portfolio of money z or e-money n. Otherwise, again, the first best may be trivially implemented. The lack of record-keeping can be interpreted as some off-line payment systems where the issuer cannot trace agents' balances in real time. Agents thus report their portfolio of money and e-money to the e-money issuer, and the e-money issuer makes transfers based on the report. In this section, we are interested in an economy where the money mechanism takes as given an exogenous money growth rate $\mu \geq 1$ (with lump-sum money transfers given by $T = (\mu - 1)(z_b + z_s) \geq 0$). This assumption simplifies the political economy, since there is no strategic interaction between the money issuer and the e-money issuer.[14] Moreover, we assume that the e-money issuer has to follow the same growth rate μ of money. This is to capture the fact that real-world e-money products often involve this feature (e.g. denominated in cash or convertible at par), and the fact that the e-money issuer, private or public, often takes inflation as given, and sets the supply to be accommodative to the e-money demand. Note that removing this restriction will only make the e-money issuer more powerful, and strengthen our conclusion.

E-money mechanism

Next we introduce the first payment technology: an e-money issuer is able to restrict participation in an e-money payment system. Specifically, the e-money issuer has the

[14] In a companion paper, we study the equilibrium interaction between money and e-money in a related environment.

power to exclude agents from holding e-money in the DM, before or after trades. Notice that the e-money issuer still does not know or control how e-money is used by agents. Furthermore, the usage of money is unrestricted.

Again, suppose the e-money issuer conducts an intervention at the end of the CM. We use mechanism design to characterize the optimal mechanism. Let $e_j = 1$ indicate that a type $j = b, s$ agent chooses to participate in the e-money mechanism. Any agent choosing $e_j = 0$ will be excluded from the e-money system and cannot hold any e-money balances in the next DM. Since agents are anonymous, this penalty can last only one period. An e-money mechanism $\mathcal{M}_E$ consists of two e-money transfer functions based on the portfolio reported, denoted as $\mathcal{M}_E = \{T_b(z,n), T_s(z,n)\}$.

CM and DM decision

In the DM, the buyer's value function under an e-money mechanism $\mathcal{M}_E$, $V_b(z,n,e_b)$, is

$$\begin{aligned} V_b(z,n,e_b) &= \alpha e_b e_s \{U[q(z+n)] + W_b[z+n-d(z+n)]\} \\ &\quad + \alpha(1-e_s e_b)\{U[q(z)] + W_b[z+n-d(z)]\} \\ &\quad + (1-\alpha) W_b(z+n). \end{aligned} \tag{16}$$

Here, since z and n are already denominated in the unit of the numeraire, the continuation value in the CM, $W_b(a)$, depends only on the sum of the real balances of money $\widetilde{z}$ and e-money $\widetilde{n}$, that is, $a \equiv \widetilde{z} + \widetilde{n}$. The first term captures the case when there is a trading opportunity, and both the buyer and the seller participate in the e-money system (i.e. $e_b = e_s = 1$). In this case, the buyer can use the total real balances a to finance the trade. The second term captures the case when there is a trading opportunity, but at least one of them does not participate in the e-money system (i.e. $e_b e_s = 0$). In this case, the buyer can only use the money z to finance the trade. The third term captures the case when there is no trading opportunity. The CM value function of the buyer under an e-money mechanism $\mathcal{M}_E$ is given by

$$W_b(a) = a + \max_{z,n,\widehat{z},\widehat{n},e_b \in \{0,1\}} \{-\mu(z+n) + e_b T_b(\widehat{z},\widehat{n}) + \beta V_b(z,n,e_b)\}, \text{ s.t.} \tag{17}$$

$$\widehat{z} \leq z, \widehat{n} \leq n.$$

Again, the constraint restricts that the buyer cannot over-report $\widehat{z}, \widehat{n}$. In the DM, the value function, $V_s(a, e_s)$, of a seller under the e-money mechanism $\mathcal{M}_E$, is

$$\begin{aligned} V_s(a, e_s) &= \alpha e_s e_b \{-C[q(z_b + n_b)] + W_s[a + d(z_b + n_b)]\} \\ &\quad + \alpha(1 - e_s e_b)\{-C[q(z_b)] + W_s[a + d(z_b)]\} \\ &\quad + (1 - \alpha) W_s(a). \end{aligned} \tag{18}$$

Similarly, if $e_b e_s = 1$, then the buyer can use $z_b + n_b$ to finance the trade. If $e_b e_s = 0$, then the buyer can only use z_b to finance the trade. In the CM, the seller's value function under an e-money mechanism $\mathcal{M}_E$, $W_s(a)$, is

$$W_s(a) = a + \max_{z,n,\widehat{z},\widehat{n},e_s \in \{0,1\}} \{-\mu(z + n) + e_s T_s(\widehat{z}, \widehat{n}) + (1 - e_s)\beta V_s(z + n, e_s)\}, \text{ s.t.} \tag{19}$$

$$\widehat{z} \leq z, \widehat{n} \leq n.$$

Incentive-compatibility for buyers

It is straightforward to establish that the bargaining solution under an e-money mechanism $\mathcal{M}_E$ is still characterized by Lemma 1. Using the linearity of $W_b(a)$, and ignoring the constant terms, one can reformulate the buyer's problem in the CM under $e_s = 1$ as

$$\max_{e_b \in \{0,1\}, z, \widehat{z}, \widehat{n}, q} \{-[\mu - \beta(1 - \alpha)] D(q) + e_b T_b(\widehat{z}, \widehat{n}) + \beta\alpha U(q)\}, \text{ s.t.} \tag{20}$$

$$\widehat{z} \leq z, \widehat{n} \leq D(q) - z.$$

As before, if $e_b = 0$, the buyer brings $\mu D(q)$ money balances to buy q in the DM. If the buyer chooses $e_b = 1$ and intends to have a post-transfer portfolio of (z, n), the buyer needs to obtain total balances $\mu D(q) - T_b(\widehat{z}, \widehat{n})$ from the CM, report $\widehat{z}$ and $\widehat{n}$, and then buy q in the following DM.

Definition 6 *An allocation (q, z_b, z_s, n_b, n_s) is incentive compatible for buyers under an e-money mechanism $\mathcal{M}_E$ if $e_b = 1$, $\widehat{z} = z = z_b$, $\widehat{n} = n_b$ and $q = q(z_b + n_b)$ solves (20) given $e_s = 1$.*

To induce buyers to participate in the mechanism, it is necessary to have an incentive-compatible allocation (q, z_b, z_s, n_b, n_s) satisfying

$$T_b(z_b, n_b) - [\mu - \beta(1-\alpha)] D(q) + \beta\alpha U(q) \geq \max_{q'} \{ - [\mu - \beta(1-\alpha)] D(q') + \beta\alpha U(q') \}. \tag{21}$$

Here, the LHS captures the payoff for joining the e-money mechanism, and the RHS captures the payoff for skipping it.

Incentive-compatibility for sellers

Similarly, using the linearity of $W_s(a)$, and ignoring the constant terms, one can reformulate the seller's problem in the CM under $e_b = 1$ as

$$\max_{e_s \in \{0,1\}, \widehat{z}, z, \widehat{n}, n} \left\{ \begin{array}{c} -\mu(z+n) + (1-e_s)\beta[z+n+\alpha[d(z_b) - C[q(z_b)]]] \\ +e_s T_s(\widehat{z}, \widehat{n}) + e_s\beta[z+n+\alpha[d(z_b+n_b) - C[q(z_b+n_b)]]] \end{array} \right\}, \text{ s.t.} \tag{22}$$

$$\widehat{z} \leq z, \widehat{n} \leq n.$$

Again, a seller joining the e-money mechanism (i.e. $e_s = 1$) has to bring extra balances to pay for the transfer $T_s(\widehat{z}, \widehat{n})$.

Definition 7 *An allocation (q, z_b, z_s, n_b, n_s) is incentive compatible for sellers under an e-money mechanism $\mathcal{M}_E$ if $e_s = 1$, $\widehat{z} = z = z_s$, $\widehat{n} = n = n_s$ solve (22) under $e_b = 1$.*

To induce sellers to participate in the mechanism, it is necessary to have an incentive-compatible allocation (q, z_b, z_s, n_b, n_s) satisfying

$$T_s(z_s, n_s) - (\mu - \beta)(z_s + n_s) + \beta\alpha[D(q) - C(q)] \geq \beta\alpha\{d(z_b) - C[q(z_b)]\}, \tag{23}$$

where the LHS captures the payoff for participating in the e-money mechanism, and the RHS captures the payoff for skipping it.

E-money issuer's budget constraint

An e-money issuer has to balance its budget, or self-finance:

Definition 8 *An e-money mechanism* $\mathcal{M}_E = \{T_b(z,n), T_s(z,n)\}$ *is self-financed with limited participation under the allocation* (q, z_b, z_s, n_b, n_s) *if*

$$T_b(z_b, n_b) + T_s(z_s, n_s) = (\mu - 1)(n_b + n_s). \tag{24}$$

Here, the e-money issuer finances transfers to buyers and sellers by issuing e-money balances.

Implementability of first best

Definition 9 *An e money mechanism* $\mathcal{M}_E$ *implements the first best with limited participation if*

a. there exists (z_b, z_s, n_b, n_s) *such that the first-best allocation* $(q^*, z_b, z_s, n_b, n_s)$ *is incentive compatible for buyers and sellers; and*

b. $\mathcal{M}_E$ *is self-financed with limited participation under the first-best allocation* $(q^*, z_b, z_s, n_b, n_s)$.

Define $\Theta(\theta)$ as the solution to

$$-(1-\beta)U(q^*) + [\beta\alpha + (1-\beta)\theta][U(q^*) - C(q^*)] = \max_q \{\beta\alpha U(q) - [\Theta - \beta(1-\alpha)]D(q)\},$$

and set $\Theta(\theta) = \infty$ if a solution $\Theta \geq \beta$ does not exist. The following proposition establishes the condition under which the first best can be achieved by an optimal e-money mechanism with limited participation.

Proposition 4 *There exists an e-money mechanism* $\mathcal{M}_E$ *that implements the first best with limited participation if and only if* $\mu \geq \Theta(\theta)$.

Proof. See the appendix. ■

This proposition shows that, to implement the first best using this e-money mechanism, buyers' bargaining power and inflation need to satisfy $\mu \geq \Theta(\theta)$. An increase in μ facilitates the implementation of first best because it reduces the outside option of non-participants who use money only as their means of payment.[15] However, an increase in θ has two opposite effects. On the one hand, it helps achieve the first best

[15] Interestingly, this is consistent with a popular view that inflation induces agents to adopt some e-money products. For example, Bitcoin is considered by some to be a safe haven from inflation.

because the holdup problem is less severe and thus buyers have higher incentives to bring the right e-money balances. On the other hand, it increases the outside option of non-participants who also face a less severe holdup problem when using money. But, in general, we know that $\Theta(0) = \infty$. Therefore, by continuity of Θ, the first best is not implementable for θ too low, which will be discussed in the following section.

Essentiality of limited participation

Proposition 5 *If there exists a money mechanism $\mathcal{M}$ that implements the first best with μ, then there also exists an e-money mechanism $\mathcal{M}_E$ that implements the first best with limited participation under the same μ.*

Proof. Since $\mathcal{M}$ implements the first best, from the proof of Proposition 2 it is necessary to have

$$\begin{aligned} 0 &\leq -[1-\beta(1-\alpha)]D(q^*) + \beta\alpha U(q^*) - \max_q \{-[\mu-\beta(1-\alpha)]D(q) + \beta\alpha U(q)\} \\ &= -(1-\beta)D(q^*) + \beta\alpha\theta[U(q^*) - C(q^*)] - \max_q \{-[\mu-\beta(1-\alpha)]D(q) + \beta\alpha U(q)\} \\ &\leq -(1-\beta)U(q^*) + [\beta\alpha + (1-\beta)\theta][U(q^*) - C(q^*)] \\ &\quad - \max_q \{-[\mu-\beta(1-\alpha)]D(q) + \beta\alpha U(q)\}. \end{aligned}$$

Thus we have $\mu \geq \Theta(\theta)$. Then, from Proposition 4 there exists an e-money mechanism $\mathcal{M}_E$ that implements the first best. ■

This proposition implies that, fixing the money growth rate, an optimal e-money mechanism featuring limited participation is at least as good as an optimal money mechanism in implementing the first-best allocation. This result may not hold in general when the e-money mechanism has to operate under a money growth rate different from that associated with the optimal money mechanism. Define

$$\underline{\theta} \equiv \frac{[1-\beta(1-\alpha)]\bar{\theta} - \beta\alpha}{1-\beta}.$$

The following proposition gives conditions under which the e-money mechanism can outperform a money mechanism.

Proposition 6 (Essentiality of e-money with limited participation) *If $\theta \in [\underline{\theta}, \bar{\theta})$, then the first best allocation*

(i) cannot be implemented by any money mechanism;

(ii) can be implemented by an e-money mechanism with limited participation under some μ.

Proof. Omitted here. ■

This proposition establishes the essentiality of e-money featuring limited participation. Part (i), implied by Proposition 2, states that no money mechanism can implement the first best when $\theta < \overline{\theta}$. At the risk of being repetitive, we reproduce here the intuition: a money mechanism uses a non-linear transfer scheme to induce buyers to "co-operate" and to carry sufficient money balances. This scheme relies on the "punishment" of eroding deviating buyers' DM trade surplus by not giving them a transfer. However, the power of this scheme is limited by the size of the buyers' trade surplus, which depends on θ. When $\theta < \overline{\theta}$, the buyers' trade surplus is insufficient for inducing them to carry the right money balances. In the extreme case of $\theta \to 0$, buyers have no surplus to be extracted.

The ability of the e-money issuer to limit participation provides an additional tool. By threatening to exclude agents from participating in the e-money system, the issuer can extract extra resources (especially from sellers), and use those extra resources to induce buyers to bring the right money balances. How much resources can be extracted from buyers and sellers? The answer is equal to the difference between the trade surplus of an e-money user and that of a money user. The power of this scheme is maximized when money users' trade surplus is zero, and this will happen when $\mu \geq \bar{\mu}$ (from Proposition 1). In this case, the threat to exclude deviators allows the issuer to extract the whole of the (discounted) trade surplus, which equals to $\beta\alpha[U(q^*) - C(q^*)]$. This explains why e-money featuring limited participation is essential.

The power of this scheme is still insufficient to achieve first best when θ is too low (i.e. $\theta \leq \underline{\theta}$). To illustrate by an extreme example, suppose that $\mu = 1$ and $\theta \to 0$. In this case, the buyer has no bargaining power and thus the price for q^* is $D(q^*) = U(q^*)$. So to induce buyers to bring $D(q^*)$ in the previous CM, the transfer T_b has to be sufficiently negative that they still have a positive payoff:

$$T_b - [\mu - \beta(1-\alpha)]D(q^*) + \beta\alpha U(q^*) - \max_q \{-[\mu - \beta(1-\alpha)]D(q) + \beta\alpha U(q)\} \geq 0.$$

However, the above discussion implies that the maximum transfer the e-money issuer

can make is $T_b^* = \beta\alpha[U(q^*) - C(q^*)]$, which is the whole of the (discounted) trade surplus. Plugging $\mu = 1$, $D(q) = U(q)$ and $T_b^* = \beta\alpha[U(q^*) - C(q^*)]$ into the LHS, the buyers' payoff becomes

$$\begin{aligned} & -[1-\beta(1-\alpha)]U(q^*) + \beta\alpha[U(q^*) - C(q^*)] + \beta\alpha U(q^*) \\ = \; & -(1-\beta)[U(q^*) - C(q^*)] + \{\beta\alpha U(q^*) - [1-\beta(1-\alpha)]C(q^*)\} \geq 0. \end{aligned}$$

As $\beta\alpha U(q^*) - [1-\beta(1-\alpha)]C(q^*) \to 0$, the LHS becomes negative, which is a contradiction. Therefore, this example shows that, when θ is small and $\beta\alpha U(q^*) - [1-\beta(1-\alpha)]C(q^*)$ is small (but remains positive, as assumed), the first-best allocation is not implementable by any e-money mechanism with limited participation. This explains part (ii) of the above proposition.

Overall, Propositions 2 and 6 characterize the implementability of first best using the money mechanism and e-money mechanism. When buyers' bargaining power is high ($\theta \geq \bar{\theta}$), an optimal money mechanism can implement the first best. Hence, e-money is not essential relative to money in this region. When buyers' bargaining power is moderate ($\bar{\theta} > \theta \geq \underline{\theta}$), only e-money featuring limited participation can implement the first best, given sufficiently high money growth μ. Hence, e-money is essential relative to money in this region. Finally, when buyers' bargaining power is too low ($\underline{\theta} > \theta$), even an e-money featuring limited participation may not implement the first best.[16]

Characterization of optimal e-money mechanism

After establishing the essentiality of e-money, we next characterize the optimal e-money mechanism.

Proposition 7 *Given μ, if there exists an e-money mechanism with limited participation $\mathcal{M}_E = \{T_b(z,n), T_s(z,n)\}$, but not any money mechanism $\mathcal{M} = \{T_b(z), T_s(z), \mu\}$, that implements the first best, then $T_s(z,n) < 0$ and $T_b(z,n) > 0$.*

Proof. See the appendix. ■

[16] Notice that e-money may remain essential in this region. Even though it cannot implement the first best, it may still improve the allocation.

As mentioned above, when θ is too low, extracting trade surplus from buyers alone cannot raise enough resources to support the first best. The power of limited participation helps implement the first best by extracting surplus from sellers (i.e. $T_s(z,n) < 0$) to cross-subsidize buyers' holding of e-money balances (i.e. $T_b(z,n) > 0$). The key benefit of limiting participation is allowing cross-subsidization from sellers to buyers, which is infeasible under a money mechanism.

Simple examples

Suppose that $\alpha = 1, \mu > \underline{\mu}$ and $\theta \in [\underline{\theta}, \overline{\theta})$. So according to Proposition 6, e-money with limited participation is essential. We will illustrate examples of simple direct and indirect mechanisms. In these extreme examples, sellers get zero trade surplus, but more general cases can be similarly constructed.

(i) Direct mechanism

Under this simple mechanism, the transfer function for sellers is a fixed fee:

$$T_s(z_s, n_s) = -\beta[D(q^*) - C(q^*)] \text{ for any } (z_s, n_s),$$

and the transfer function for buyers is

$$T_b(z_s, n_b) = \begin{cases} (\beta + \mu - 1)D(q^*) - \beta C(q^*), \text{ if } n_b = D(q^*) \\ 0, \text{ otherwise.} \end{cases}.$$

When $\mu > \underline{\mu}$, Proposition 1 implies that buyers not joining the e-money mechanism will choose not to trade. In this case, a buyer joining the e-money mechanism needs to bring $\beta C(q^*) + (1-\beta)d^*$ from the CM, receive a transfer T_b from the issuer, and bring $D(q^*)$ into the DM to consume q^*. We can show that the participation constraint is satisfied when

$$\theta \geq \underline{\theta} = \frac{\overline{\theta} - \beta}{1 - \beta}.$$

Notice that this scheme exhibits the features of non-linear transfers and cross-subsidization.

(ii) Indirect mechanism: fixed membership fee and proportional rewards

The e-money issuer imposes a fixed membership fee B_b on buyers, who can then

collect interest on their money balances at the rate R in the end of the CM:

$$\begin{aligned} R &= \frac{\mu}{\beta} - 1, \\ T_b &= (2\beta - 1)\, D\,(q^*) - \beta C(q^*). \end{aligned}$$

Without paying B_b, a buyer cannot use e-money in the next DM. Similarly, in order to receive e-money in the next DM, a seller has to pay

$$T_s = -\beta[D(q^*) - C(q^*)].$$

The e-money issuer's budget is balanced. Obviously, sellers are indifferent between joining or not. Buyers have an incentive to join when

$$T_b - D(q^*)\beta + \beta U(q^*) \geq 0,$$

where $\beta D(q^*)$ is the balance they need to bring to the DM so that, after the interest payment, they have real balance $D(q^*)$ to finance the efficient quantity in the DM. One can show that this is positive when $\theta \geq \overline{\theta}$. Notice that this scheme also exhibits the features of piecewise linear transfers and cross-subsidization. This mechanism does not involve money. Appendix B considers an example involving money deposits. In that example, the e-money mechanism is designed to support the positive value of money in equilibrium.

Summary

In this section, we learned that:

1. When buyers have moderate bargaining power and the money growth rate is high, an e-money mechanism with limited participation is essential to implement the first best.

2. Given the money growth rate, an optimal e-money mechanism with limited participation is always more essential than any money mechanism; i.e. there are scenarios where the former, rather than the latter, can achieve the first best. In this case, cross-subsidization from sellers to buyers is an essential feature to implement the first best.

3. The first best can be implemented by a simple indirect mechanism with fixed membership fees on buyers and sellers, and proportional rewards on buyers' balances.

5 Electronic Money with Limited Transferability

Next we consider limited transferability as an alternative feature of e-money. Suppose that the e-money issuer can no longer limit the participation of users, but has instead the power to block e-money transfers among agents in the DM. However, the e-money issuer again does not have any record-keeping technology, and therefore does not know the amount of e-money transferred or the true identities of the payer and payee. In the DM, a payer chooses whether to pay the e-money issuer Δ_b units of e-money in order to transfer e-money to someone else. If this fee is not paid, then the e-money transfer will be blocked by the issuer. Similarly, a payee chooses whether to pay the e-money issuer Δ_s units of e-money in order to receive e-money from someone else. Otherwise, the transfer is blocked. We interpret Δ_b and Δ_s as interchange fees. We focus on the case $\Delta_b + \Delta_s \geq 0$, in which otherwise buyers and sellers can fake DM trades (by sending and receiving an arbitrary small amount of e-money) to earn the transfer $\Delta_b + \Delta_s$ from the e-money issuer. Notice that limited transferability is different from limited participation: in a mechanism with limited participation, agents need to pay fees to join the e-money mechanism, in order to use e-money in the DM; in a mechanism with limited transferability, agents first match in the DM and then decide whether to pay the interchange fees for using e-money as the means of payment, regardless of whether they have joined the e-money mechanism in advance. An important distinction is that a mechanism featuring limited participation collects fees only in the CM, while a mechanism featuring limited transferability can also collect fees in the DM. This distinction leads to different abilities to implement the first-best allocation.[17] In the CM, e-money is assumed to be freely transferable among agents.[18]

CM and DM decision

[17] Note that the money issuer does not need to be physically present in a bilateral match in order to collect the interchange fees. Instead, in the absence of a zero-sum constraint, part of the balances transferred can be destroyed automatically in the payment process.

[18] Allowing the issuer to have the additional power to restrict transferability in the CM can only strengthen our findings.

An e-money mechanism $\mathcal{M}_L$ featuring limited transferability consists of the e-money transfer functions based on the portfolio reported and fees in the DM, denoted as $\mathcal{M}_L = \{\Delta_b, \Delta_s, T_b(z,n), T_s(z,n)\}$. In the DM, the buyer's value function under an e-money mechanism $\mathcal{M}_L$, $V_b(z,n)$, is

$$\begin{aligned} V_b(z,n) &= \alpha\left[U\left[q(z,n)\right] + W_b\left[z+n-d_z(z,n)-d_n(z,n)-I_{d_n(z,n)>0}\Delta_b\right]\right] \\ &\quad + (1-\alpha)W_b(z+n), \end{aligned} \tag{25}$$

where $\{q(z,n), d_z(z,n), d_n(z,n)\}$ is the bargaining solution as a function of the buyer's portfolio. It specifies that the buyer needs to pay d_z units of real money balances and d_n units of real e-money balances for q units of the DM goods produced by the seller. $I_{d_n(z,n)>0}$ is an indicator function capturing the fact that the buyer needs to pay the fee Δ_b whenever e-money payment is positive, or $d_n(z,n) > 0$.

$W_b(a)$ is the CM value function of the buyer under an e-money mechanism $\mathcal{M}_L$, which is given by

$$\begin{aligned} W_b(a) &= a + \max_{\substack{z,\widehat{z},n,\widehat{n} \\ e_b\in\{0,1\}}} \left\{-\mu(z+n) + e_b T_b^1(\widehat{z},\widehat{n}) + \beta V_b(z,n)\right\}, \\ &\quad \text{s.t. } \widehat{z} \le z,\ \widehat{n} \le n. \end{aligned} \tag{26}$$

Unlike in the case of limited participation, the DM value function does not depend on the choice of e_b, because the e-money issuer cannot restrict the buyer's e-money transfer conditional on e_b.

In the DM, the value function, $V_s(a)$, of a seller who has joined an e-money mechanism $\mathcal{M}_L$ is

$$\begin{aligned} V_s(a) &= \alpha\left[-C\left[q(z_b,n_b)\right] + W_s\left[a + d_z(z_b,n_b) + d_n(z_b,n_b) - I_{d_n(z_b,n_b)>0}\Delta_s\right]\right] \\ &\quad + (1-\alpha)W_s(a). \end{aligned} \tag{27}$$

Again, the seller needs to pay a fee Δ_s in order to receive the e-money payment $d_n(z_b,n_b) > 0$.

In the CM, the seller's value function under an e-money mechanism $\mathcal{M}_L$, $W_s(a)$, is

$$W_s(a) = \max_{\substack{z,\widehat{z},n,\widehat{n} \\ e_s \in \{0,1\}}} \{a - \mu(z+n) + e_s T_s(\widehat{z},\widehat{n}) + \beta V_s(z+n)\}, \quad (28)$$
$$\text{s.t. } \widehat{z} \leq z,\ \widehat{n} \leq n.$$

Bargaining solution

Given the terms of trade (q, d_z, d_n), the DM trade surpluses of the buyer and the seller are, respectively,

$$\begin{aligned} S_b(q,d_z,d_n) &= U(q) + W_b(z_b + n_b - d_z - d_n - I_{d_n>0}\Delta_b) - W_b(z_b+n_b), \\ S_s(q,d_z,d_n) &= -C(q) + W_s(a_s + d_z + d_n - I_{d_n>0}\Delta_s) - W_s(a_s) \\ &= -C(q) + W_s(d_z + d_n - I_{d_n>0}\Delta_s) - W_s(0). \end{aligned}$$

By linearity of W_s, the surplus does not depend on the seller's balances a_s. Under an e-money mechanism $\mathcal{M}_L$, the bargaining solution $\{q(z_b,n_b), d_z(z_b,n_b), d_z(z_b,n_b)\}$ is given by

$$\max_{q,d_z,d_n} S_b(q,d_z,d_n) + S_s(q,d_z,d_n), \text{ s.t.} \quad (29)$$

$$\begin{aligned} S_b(q,d_z,d_n) &= \theta[S_b(q,d_z,d_n) + S_s(q,d_z,d_n)], \\ d_z &\leq z_b, \quad (30) \\ d_n + I_{d_n>0}\Delta_b &\leq n_b, \quad (31) \\ I_{d_n>0}\Delta_s &\leq d_n. \quad (32) \end{aligned}$$

Here, the first constraint is the pricing protocol. The second and third are, respectively, the buyer's liquidity constraints on money and e-money payments. The last constraint requires that the seller's e-money balance received be sufficient for financing the interchange fee imposed on the seller. Here, we assume that sellers cannot use their own balances to pay the fee.

Using the fact that $W_b(a)$ and $W_s(a)$ are linear, the bargaining problem (29) can be reformulated as

$$\max_{q,d_z,d_n} \{U(q) - C(q) - I_{d_n>0}(\Delta_b + \Delta_s)\}, \text{ s.t. (30), (31), (32), and} \quad (33)$$

$$(1-\theta)\left[U(q)-d_z-d_n-I_{d_n>0}\Delta_b\right]=\theta\left[-C(q)+d_z+d_n-I_{d_n>0}\Delta_s\right].$$

Define $\Delta \equiv \Delta_b + \Delta_s$. It is straightforward to show the following lemma, which characterizes the bargaining solution under $\mathcal{M}_L$.

Lemma 2 *The bargaining solution* $\{q(z_b,n_b), d_z(z_b,n_b), d_n(z_b,n_b)\}$ *satisfies:*

$$d_z = \min\{z_b, (1-\theta)U^* + \theta C^*\},$$

$$d_n + I_{d_n>0}\Delta_b = \begin{cases} \min\{n_b, (1-\theta)U^* + \theta C^* + \theta\Delta - d_z\}, \\ \quad \text{if } n_b \geq \Delta \text{ and } U(q) - C(q) - \Delta \geq U(q(z_b)) - C(q(z_b)) \\ 0, \text{ otherwise} \end{cases}$$

$$D(q) = d_z + d_n + I_{d_n>0}(\Delta_b - \theta\Delta).$$

The key difference from the bargaining solution in the previous section is that the buyer and the seller now have to share the total interchange fee Δ according to their bargaining weights. In the presence of interchange fees, there is a pecking order of payment: using money before e-money to avoid paying the interchange fees. The interchange fees partially pass through to prices. From Lemma 2 the total payment made by the buyer is given by $d_z + d_n + I_{d_n>0}\Delta_b = D(q) + I_{d_n>0}\theta\Delta$, and the total payment received by the seller is given by $d_z + d_n - I_{d_n>0}\Delta_s = D(q) - I_{d_n>0}(1-\theta)\Delta$. A higher buyer's bargaining power θ will result in a higher pass-through of the interchange fees Δ on the total payment made by the buyer.

Incentive-compatibility for buyers

Using the linearity of $W_b(a)$, and ignoring the constant terms, one can reformulate the buyer's problem in the CM as

$$\max_{\substack{\widehat{z},x,\widehat{n},n,\\ e_s\in\{0,1\}}} \left\{ \begin{array}{l} -(\mu-\beta)(z+n) + e_b T_b(\widehat{z},\widehat{n}) \\ +\beta\alpha\left[U\left[q(z,n)\right] - d_z(z,n) - d_n(z,n) - I_{d_n>0}\Delta_b\right] \end{array} \right\}, \tag{34}$$
$$\text{s.t. } \widehat{z} \leq z,\ \widehat{n} \leq n.$$

Definition 10 *An allocation* (q, z_b, z_s, n_b, n_s) *is incentive compatible for buyers under an e-money mechanism* $\mathcal{M}_L$ *if* $e_b = 1$, $\widehat{z} = z_b$, $\widehat{n} = n_b$, *and* q *solves (34).*

Notice that a buyer who chooses to skip the mechanism $\mathcal{M}_L$ in CM (i.e., $e_b = 0$) will not bring e-money to the DM, because money and e-money have the same inflation rate μ, but using e-money involves an extra interchange fee Δ_b, as shown in Lemma 2. Thus, $e_b = 0$ implies $n_b = d_n = 0$. As before, to induce buyers to participate in the mechanism, it is necessary to have an incentive-compatible allocation (q, z_b, z_s, n_b, n_s), where $e_b = 1$ and $d_n > 0$, satisfies

$$T_b(z_b, n_b) - [\mu - \beta(1-\alpha)][D(q) + \theta\Delta] + \beta\alpha U(q) \geq \max_{q'} \{-[\mu - \beta(1-\alpha)] D(q') + \beta\alpha U(q')\}. \tag{35}$$

Incentive-compatibility for sellers

Similarly, using the linearity of $W_s(a)$, and ignoring the constant terms, one can reformulate the seller's problem in the CM as

$$\max_{\substack{\widehat{z},z,\widehat{n},n,\\ e_s\in\{0,1\}}} \{(\beta-\mu)(z+n) + e_s T_s(\widehat{z},\widehat{n})\}, \tag{36}$$
$$\text{s.t. } \widehat{z} \leq z,\ \widehat{n} \leq n.$$

Definition 11 *An allocation (q, z_b, z_s, n_b, n_s) is incentive compatible for sellers under an e-money mechanism $\mathcal{M}_E$ if $e_s = 1$, $\widehat{z} = z = z_s$, $\widehat{n} = n = n_s$ solve (36).*

To induce sellers to participate in the mechanism, it is necessary to have an incentive-compatible allocation (q, z_b, z_s, n_b, n_s) such that $e_s = 1$ and it satisfies

$$-(\mu - \beta)(z_s + n_s) + T_s(z_s, n_s) \geq 0. \tag{37}$$

E-money issuer's budget constraint

Definition 12 *An e-money mechanism $\mathcal{M}_L \equiv \{\Delta_b, \Delta_s, T_b(z,n), T_s(z,n)\}$ is self-financed with limited transferability under the allocation (q, z_b, z_s, n_b, n_s) if*

$$T_b(z_b, n_b) + T_s(z_s, n_s) = \alpha(\Delta_b + \Delta_s) + (\mu - 1)(n_b + n_s). \tag{38}$$

Implementability of first best

Definition 13 *An e-money mechanism $\mathcal{M}_L$ implements the first best with limited transferability if*

a. there exists (z_b, z_s, n_b, n_s) such that the first best allocation $(q^*, z_b, z_s, n_b, n_s)$ is incentive compatible to buyers and sellers; and

b. $\mathcal{M}_L$ is self-financed with limited transferability under the first-best allocation $(q^*, z_b, z_s, n_b, n_s)$.

As before, define $\Phi(\theta)$ as the solution to

$$(\beta + \alpha - 1) U(q^*) - \alpha C(q^*) = \max_q \{\beta\alpha U(q) - [\Phi - \beta(1-\alpha)] D(q)\},$$

and set $\Phi(\theta) = \infty$ if a solution $\Phi \geq \beta$ does not exist. Notice that $\Phi(\theta)$ is increasing in θ for all finite $\Phi(\theta)$. The following proposition establishes when the optimal e-money mechanism featuring limited transferability is efficient.

Proposition 8 *There exists some μ and an e money mechanism $\mathcal{M}_L$ that implements the first best with limited transferability if and only if either (a) $\theta \geq \bar{\theta}$ or (b) $(\beta + \alpha - 1) U(q^*) > \alpha C(q^*)$. If (a) does not hold, then there exists $\mathcal{M}_L$ implementing the first best if and only if $\mu \geq \Phi(\theta)$.*

Proof. See the appendix. ■

This proposition shows that, to implement the first best using this e-money mechanism, buyers' bargaining power and inflation need to satisfy $\mu \geq \Phi(\theta)$. It is straightforward to show that $\Phi(\theta)$ is increasing in θ. The idea is that an increase in θ raises the value of the buyers' outside option of non-participation. Higher inflation is needed to induce them to join the mechanism. Therefore, e-money featuring limited transferability can implement the first best when inflation is not too low. In particular, when $\alpha = 1$, condition (b) in Proposition 8 is always satisfied due to (15). In this case, the first-best allocation can be achieved for any θ, when the inflation rate is sufficiently high.

Essentiality of limited transferability

We first derive conditions under which an optimal mechanism with limited transferability is at least as good as one with limited participation, and vice versa.

Proposition 9 *(a) Suppose that $\theta \leq \alpha$. If there exists an e-money mechanism $\mathcal{M}_E$ that implements the first best with limited participation, then there also exists an e-money mechanism $\mathcal{M}_L$ that implements the first best with limited transferability under the same μ.*

(b) Suppose that $\theta > \alpha$. If there exists an e-money mechanism $\mathcal{M}_L$ that implements the first best with limited transferability, then there also exists an e money mechanism $\mathcal{M}_E$ that implements the first best with limited participation under the same μ.

Proof. See the appendix. ■

Proposition 9 gives a sharp condition $\theta \leq \alpha$ that is sufficient and necessary for limited transferability to be at least as essential as limited participation. Before interpreting this condition, we want to check whether there are situations where limited transferability is strictly more essential than limited participation, and vice versa. The answer is yes, for both. For example, when $\alpha = 1$, the condition in part (a) of Proposition 9 is satisfied. In this case, fixing the money growth rate, an optimal e-money mechanism featuring limited transferability is always at least as good as an optimal money mechanism in implementing the first-best allocation. More generally, combining Propositions 4, 8 and 9, we have the following result.

Proposition 10 (Essentiality of e-money with limited transferability) *(a) If $\alpha = 1$ and $\theta < \underline{\theta}$, then the first best allocation can be implemented by an e-money mechanism with limited transferability when $\mu \geq \bar{\mu}$. The first best allocation cannot be implemented by any e-money mechanism with limited participation;*

(b) If $\theta \in (\alpha, \theta)$ and $(\beta + \alpha - 1) U(q^*) < \alpha C(q^*)$, then the first best allocation can be implemented by an e-money mechanism with limited participation when $\mu \geq \bar{\mu}$. The first-best allocation cannot be implemented by any e-money mechanism with limited transferability.

Proof. Omitted here. ■

This proposition provides some parameter regions in which there are different orders of essentiality of e-money technologies. In general, limited transferability is more powerful than limited participation under low θ and high α. The opposite is true under high θ and low α. What is the intuition? On the one hand, the amount of interchange fees passed through to the buyer is $\theta\Delta$, so a low value of θ means that the buyer bears

a small interchange fee burden. On the other hand, recall that limited participation allows the issuer to use exclusion from period $t+1$ DM as a threat to enforce fees in period t. That is why the maximum surplus extractable from a seller, $\alpha\beta[D(q^*)-C(q^*)]$, is discounted, since the fee is paid a period in advance. In contrast, the ability to limit transferability allows an issuer to extract the seller's trade surplus in period t DM by enforcing interchange fees in the same period. The maximum surplus extractable from a seller becomes $\alpha[D(q^*)-C(q^*)]$, without discounting. Therefore, the gain from postponing fee collection, which relaxes the seller's participation constraint, is stronger when α is high. In sum, for high α the buyer can be rewarded a large sum financed by the seller's surplus; for low θ the buyer only bears a small interchange pass-through. As a result, under high α and low θ the technology limiting transferability can help induce buyers to bring sufficient balances to support the first-best allocation, which cannot be done by the technology limiting participation. But will postponing fee collection tighten the issuer's budget constraint? No, because the issuer can always create more e-money balances when needed. From the issuer's point of view, collecting the fee in the CM or in the following DM does not matter, as long as the money growth rate between two CM markets can be maintained at μ. In particular, the issuer can temporarily create extra e-money balances in CM, and undo them later when interchange fees are collected in the following DM. Therefore, limited transferability allows the e-money issuer to postpone fee collection, maximizing surplus extraction, without tightening its budget constraint.[19]

Characterization of optimal e-money mechanism

After establishing the essentiality of e-money, we now characterize the optimal e-money mechanism.

Proposition 11 *Given μ,*

(a) if there exists an e-money mechanism with limited transferability $\mathcal{M}_L = \{\Delta_b, \Delta_s, T_b(z,n), T_s(z,n)\}$, but not any e-money mechanism with limited participation $\mathcal{M}_E$, that implements the first best, then $\Delta > 0$ and $T_b(z_n, n_n) < 0$.

[19] Note that $\partial\theta/\partial\beta > 0$, implying that limited transferability is more essential relative to limited participation when the discount factor is low. A real-world interpretation is that charging interchange fees at the time of the transaction is more desirable relative to charging a membership fee in advance, when the frequency of membership fee payment is low (e.g. annual membership paid a year in advance).

(b) if there exists $\mathcal{M}_E = \{T_b(z,n), T_s(z,n)\}$ *but not any* $\mathcal{M}_L$ *that implements the first best, then* $T_s(z,n) > 0$ *and* $T_b(z_n, n_n) < 0$.

Proof. See the appendix. ■

As discussed above, to implement the first best, the issuer has to extract trade surplus in the DM ($\Delta > 0$), which is then used to induce buyers to carry sufficient e-money balances in the CM ($T_b(z_n, n_n) < 0$). Note that this scheme requires the issuer to temporarily expand the e-money supply in the CM (to pay buyers $T_b(z_n, n_n)$) and later undo it in the DM (by charging fees Δ), ensuring constant money growth across periods.

Simple examples

Suppose that $\mu > \bar{\mu}$. We will illustrate examples of simple direct and indirect mechanisms. In these extreme examples, sellers get zero trade surplus, but more general cases can be similarly constructed.

(i) Direct mechanism

Under this simple mechanism, the transfer functions are

$$\begin{aligned} T_b(z_s, n_b) &= \begin{cases} \mu d_n^* - C(q^*), \text{ if } n_b = d_n^* \\ 0, \text{ otherwise.} \end{cases} \\ T_s(z_s, n_s) &= 0, \text{ for any } (z_s, n_s), \end{aligned}$$

where $d_n^* = D(q^*) + \theta\Delta_s$, and the interchange fees are

$$\begin{aligned} \Delta_b &= 0, \\ \Delta_s &= d^* - C(q^*). \end{aligned}$$

The budget constraint of the issuer is satisfied. Obviously, the seller's participation constraint is satisfied. When $\mu > \bar{\mu}$, Proposition 1 implies that buyers not joining the e-money mechanism will choose not to trade. In this case, a buyer has an incentive to join the e-money mechanism to bring d_n^* into the DM to consume q^* if $-C(q^*) + \beta U(q^*) \geq 0$, which is always satisfied. This scheme features cross-subsidization from sellers to buyers, with non-linear pre-trade transfers to buyers, and post-trade fees on sellers.

(ii) Indirect mechanism: fixed membership fee, proportional rewards and interchange fee on merchants

The e-money issuer imposes a fixed membership fee B_b on buyers, who can then collect interest on their money balances at the rate R in the end of the CM:

$$\begin{aligned} R &= \frac{\mu}{\beta} - 1, \\ B_b &= C(q^*) - \beta d_n^*, \end{aligned}$$

where $d_n^* = D(q^*) + \theta \Delta_s$. Also, the seller has to pay an interchange fee

$$\Delta_s = d_n^* - C(q^*).$$

Obviously, sellers are indifferent between joining or not. The issuer's budget is balanced. Buyers have an incentive to join when

$$-\beta d_n^* - B_b + \beta U(q^*) \geq 0,$$

where βd_n^* is the balance they need to acquire in the CM so that, after interest payment, they have real balance d_n^* to finance the efficient quantity in the DM. One can show that this is positive when $-C(q^*) + \beta U(q^*) \geq 0$. This scheme features cross-subsidization from sellers to buyers, with piecewise linear pre-trade transfers to buyers, and post-trade fees on sellers. This mechanism does not involve money. Appendix C gives an example involving money deposits. In that example, the e-money mechanism is designed to support a positive value of money in equilibrium.

Summary

In this section, we learned that:

1. Given a money growth rate, an optimal e-money mechanism with limited transferability is always more essential than any money mechanism.

2. When buyers have low bargaining power and high frequency of trade, an e-money mechanism with limited transferability is more essential than any e-money mechanism with limited participation (and vice versa). In this case, cross-subsidization from sellers to buyers using interchange fees is an essential feature to implement the first best.

3. The first best can be implemented by a simple indirect mechanism with fixed membership fees on buyers, proportional rewards on buyers' balances, and interchange fees on sellers.

6 Extension on Competitive Pricing

The above analysis only considers an environment with bilateral trading under the pricing protocol of proportional bargaining, which is convenient for capturing buyers' and sellers' bargaining powers and two-sided externalities. One may wonder if our result is robust in other trading environments. In particular, if agents conduct monetary trades in a centralized market and take competitive prices as given, they do not consider other agents' balances and adoption decisions, and do not have any bargaining power. Even in that environment, however, the curvatures of U and C imply that buyers' and sellers' surpluses remain positive. As a robustness check, we show in the appendix that all the main results on essentiality of various money and e-money mechanisms still hold, if we reinterpret agents' bargaining powers appropriately as the relative trade surplus at the first best under competitive pricing, $\theta = [U(q^*) - C'(q^*)q^*] / [U(q^*) - C(q^*)]$.

7 Discussion and Conclusion

Using the mechanism design approach, we have identified several essential features of e-money that help improve the efficiency of a monetary economy. First, unlike conventional cash, e-money systems can exclude participation. Second, unlike cash, e-money systems can restrict and block balance transfers and these transfers are not necessarily zero-sum bilaterally. Our model then predicts that an optimally designed e-money system with the above technologies can exhibit several features, including non-linear pricing, membership fees, interchange fees and rewards to buyers. This prediction does have some empirical support, since several successful real-world e-money systems also possess these features. For example, the Octopus card sets a non-linear fee structure, imposing fixed and variable fees on merchants, and offering rewards and discounts to

consumers.[20] PayPal also charges merchants a fee on accepting payments.[21] According to our model, these pricing features are important components for incentivizing participants and cross-subsidizing across different types in order to support efficient economic outcomes.

The above implications provide useful lessons for policy-makers. First, e-money is fundamentally different from conventional money. Improved information and technologies as a result of the introduction of e-money allow more general fee structures and can increase efficiency. Second, pricing arrangements such as merchant and interchange fees can be essential components of an optimal payment system. Hence, fee regulation may distort the optimal mechanism and reduce welfare. For example, the Durbin Amendment to the Dodd-Frank Act limits the maximum permissible interchange fees for a debit card transaction based on issuers' costs associated with processing, clearance and settlement. Our theory suggests that imposing this type of regulation on e-money can be welfare reducing because the optimal fee is positive even in an environment in which the physical cost of payments is zero. Third, our theory suggests that different payment instruments emerge to mitigate different economic frictions. For example, there is a fundamental difference between money (including e-money) and credit because consumers need to acquire balances in advance in the former but not the latter case. The optimal design of a money-based payment system is different from that of a credit-based system, since they are subject to different incentive and feasibility constraints. For instance, limiting interchange fees can be optimal for some specific payment instruments but not all.

While our paper has provided novel economic and policy insights, we have abstracted from several interesting aspects. First, we did not model other potentially useful features of e-money such as convenience and transaction speed: while these features can enhance efficiency, they may not be essential for mitigating fundamental frictions in a monetary economy. Moreover, additional features of e-money can be easily incorporated into our environment. Second, we have assumed that there is no cost

[20] Merchants are subject to a fee structure involving a fixed deposit, a fixed monthly fee and a variable fee proportional to transaction value. Individual buyers need to pay a fixed deposit to obtain an Octopus card. Rewards are offered to cardholders, such as Octopus reward (at least 0.5% of spending) and discounts on selected products (e.g. transportation).

[21] Under the basic arrangement, PayPal charges a 2.9% merchant fee plus $0.30 per transaction, with volume discounts applied.

of operating an e-money system to highlight the result that the optimal fee on sellers can be positive even in this extreme setting. In a more general environment, we expect that a similar pricing arrangement would remain optimal because it could help raise resources efficiently to finance the operation of the system. Third, our model focuses on a simple pricing protocol – proportional bargaining, because it can easily capture the split of trade surplus between the buyer and the seller. As shown, our finding can be generalized to other cases, including competitive pricing, where the parameter θ can be mapped to the share of trade surplus allocated to the buyer under the first-best allocation. Fourth, we have assumed that trading status (i.e. buyers and sellers) is permanent because this is more realistic given the frequency of trade captured by the model. However, our findings will remain unchanged when types are random (especially when agents know their types before portfolio choice is made). Fifth, we have not studied the equilibrium outcome when e-money is issued or operated by private profit-maximizing agents. In a companion paper, Chiu and Wong (2014), we investigate the potential inefficiency of the market provision of e-money. Finally, our model builds on a very standard environment used in the money search literature. Many alternative model variations (such as endogenous entry and endogenous matching) can be explored, but we leave those interesting analyses for future work.

Appendix

A Proof of Proposition 2

We sketch the proof for Proposition 2 as follows. First, we show that if $\theta < \bar{\theta}$, then there does not exist any money mechanism $\mathcal{M}$ that implements the first best. Suppose that this is not the case, and denote (q^*, z_b, z_s) as the first-best allocation implemented. Since the equilibrium exists as the first-best allocation, we must have $\mu \geq \beta$. Denote $\eta \equiv (\beta - \mu) z_s + T_s(z_s)$. Since (q^*, z_b, z_s) is incentive compatible for sellers, from (13) we have $\eta \geq 0$. Substituting (14) into the definition of η, we have

$$T_b(z_b) - \mu z_b = -\eta - (1-\beta) z_s - z_b. \tag{A.1}$$

Since (q^*, z_b, z_s) is also incentive compatible for buyers, from (11) we have

$$\begin{aligned} & \max_{q'} \{ -[\mu - \beta(1-\alpha)] D(q') + \beta\alpha U(q') \}, \\ \leq\ & T_b(z_b) - [\mu - \beta(1-\alpha)] z_b + \beta\alpha U(q^*), \\ =\ & \underbrace{-\eta - (1-\beta) z_s}_{\leq 0} + (\theta - \bar{\theta}) [1 - \beta(1-\alpha)] [U(q^*) - C(q^*)] \leq 0, \end{aligned}$$

where we have substituted (A.1) and used the fact that

$$\begin{aligned} & \beta\alpha U(q^*) - [1 - \beta(1-\alpha)] D(q^*), \\ =\ & \beta\alpha U(q^*) - [1 - \beta(1-\alpha)] [(1-\theta) U(q^*) + \theta C(q^*)], \\ =\ & (\theta - \bar{\theta}) [1 - \beta(1-\alpha)] [U(q^*) - C(q^*)] < 0. \end{aligned}$$

Since we have $\max_{q'} \{ -[\mu - \beta(1-\alpha)] D(q') + \beta\alpha U(q') \} \geq 0$, there is a contradiction.

On the other hand, if $\theta \geq \bar{\theta}$, we can construct a money mechanism $\mathcal{M}$ that implements the first best. Consider the following money mechanism: $T_s(z) = 0$ for all z, $\mu = \bar{\mu}$ and

$$T_b(z) = \begin{cases} (\bar{\mu} - 1) D(q^*), \text{ if } z = D(q^*) \\ 0, \text{ otherwise.} \end{cases}$$

B Implementation by Indirect Mechanisms: Interest-Bearing Money

So far, we have exploited the power of the revelation principle and focused on the set of direct mechanisms that implements the first-best allocation when $\theta \geq \bar{\theta}$. In general, the reverse of the revelation principle is not true: it is possible to have some first-best allocations that can be implemented by a direct mechanism, such as the one constructed above, but not by indirect mechanisms. However, we will show in this section how an indirect mechanism based on the one proposed by Andolfatto (2010) can be used to implement the first best when $\theta \geq \bar{\theta}$.

As with the mechanism suggested by Andolfatto (2010), consider now the money issuer charges buyers a fixed membership fee B to collect interest on money at the rate R in the end of the CM. The mechanism has nothing to do with sellers. Thus, an Andolfatto's mechanism is indexed by a triple $\mathcal{M}_A = \{B, R, \mu\}$. The optimization problem of a buyer in the CM under an Andolfatto's mechanism can be formulated as

$$\max_{z', q, e \in \{0,1\}} \{e\left[-z' - B + \beta U\left(q\right)\right] + \left(1 - e\right)\left[-\mu D\left(q\right) + \beta U\left(q\right)\right]\}, \text{ s.t.} \tag{B.1}$$

$$D\left(q\right) = \frac{1+R}{\mu} z'. \tag{B.2}$$

In the equilibrium, we have $z_b = z'$ and $z_s = 0$.

Definition 14 *An Andolfatto's mechanism $\mathcal{M}_A = \{B, R, \mu\}$ is self financed under the allocation $(q, d, z_b, 0)$ if*

$$Rz_b = B + \left(\mu - 1\right) z_b. \tag{B.3}$$

Definition 15 *An Andolfatto's mechanism $\mathcal{M}_A$ implements the first best if*

a. $z' = \frac{\mu d^*}{1+R} = z_b$, $q = q^*$ and $e = 1$ solves (B.1); and

b. $\mathcal{M}_A$ is self-financed under the first-best allocation $(q^*, d^*, z_b, 0)$.

Define μ_0 as solution $\mu = \mu_0$ solving

$$-d^* + \beta U\left(q^*\right) = \max_{q'} \{-\mu D\left(q'\right) + \beta U\left(q'\right)\}. \tag{B.4}$$

The following lemma shows when μ_0 is well-defined.

Lemma 3 *There exists μ_0 solving (B.4) if and only if $\theta \geq \overline{\theta}$. If such μ_0 exists then $\mu_0 > 1$.*

Proof. Omitted here. ■

The following proposition characterizes the set of an Andolfatto's mechanism that implements the first best.

Proposition 12 *Suppose that $\theta \geq \overline{\theta}$. There exists an Andolfatto's mechanism that implements the first best, which is constructed as follows:*

a. $\mu \geq \mu_0$;

b. $R = \frac{\mu}{\beta} - 1$; *and*

c. $B = \mu(1-\beta)d^*$.

Proof. Omitted. ■

C Proof of Proposition 4

First, we want to show that if $\mu < \Theta(\theta)$ then there does not exist an e-money mechanism $\mathcal{M}_E$ that implements the first best with limited participation. Suppose that this is not the case. Then there exists an e-money mechanism $\mathcal{M}_E$ that implements the first best $(q^*, z_b, z_s, n_b, n_s)$. Define

$$\begin{aligned}\eta \equiv\ & T_s(z_s, n_s) - (\mu - \beta)(z_s + n_s) \\ & + \beta\alpha\left[D(q^*) - C(q^*) - d(z_b) + C\left[q(z_b)\right]\right].\end{aligned}$$

The fact that $(q^*, z_b, z_s, n_b, n_s)$ is incentive compatible for sellers and buyers implies that $\eta \geq 0$ and $D(q^*) = z_b + n_b$. Substituting (24) and $D(q^*) = z_b + n_b$ to (23), we have

$$T_b(z_b, n_b) - \left[\mu - \beta(1-\alpha)\right] D(q^*) = -\eta - A - \beta\alpha C(q^*) - (1-\beta) D(q^*), \qquad \text{(C.1)}$$

where

$$A \equiv \beta\alpha\left[d(z_b) - C\left[q(z_b)\right]\right] + (\mu - \beta) z_s + (\mu - 1) z_b + (1-\beta) n_s \geq 0.$$

Notice that the definition of $\Theta(\theta)$ implies that $\mu < \Theta(\theta)$ if and only if

$$\left|\beta\alpha(1-\theta)+\left[1-\beta(1-\alpha)\right]\left(\theta-\bar{\theta}\right)\right|\left[U(q^*)-C(q^*)\right] < \max_q\left\{-\left[\mu-\beta(1-\alpha)\right]D(q)+\beta\alpha U(q)\right\}. \tag{C.2}$$

Since $(q^*, z_b, z_s, n_b, n_s)$ is also incentive compatible for buyers, from (21) we have

$$\begin{aligned}
& \max_q\left\{-\left[\mu-\beta(1-\alpha)\right]D(q)+\beta\alpha U(q)\right\} \\
\leq\ & T_b(z_b,n_b)-\left[\mu-\beta(1-\alpha)\right]D(q^*)+\beta\alpha U(q^*) \\
=\ & -\eta-A-(1-\beta)D(q^*)+\beta\alpha\left[U(q^*)-C(q^*)\right] \\
=\ & -\eta-A+\left[\beta\alpha(1-\theta)+\left(\theta-\bar{\theta}\right)\left[1-\beta(1-\alpha)\right]\right]\left[U(q^*)-C(q^*)\right] \\
<\ & -\eta-A+\max_q\left\{-\left[\mu-\beta(1-\alpha)\right]D(q)+\beta\alpha U(q)\right\},
\end{aligned}$$

where we have substituted (C.1), (C.2) and used the fact that $D(q^*)-C(q^*) = (1-\theta)\left[U(q^*)-C(q^*)\right]$. A contradiction.

On the other hand, if $\mu \geq \Theta(\theta)$, we can construct an e-money mechanism $\mathcal{M}_E$ that implements the first best with limited participation. Since $\mu \geq \Theta(\theta)$, we have $\varepsilon_0 = -(1-\beta)D(q^*)+\beta\alpha\left[U(q^*)-C(q^*)\right]-\max_q\left\{-\left[\mu-\beta(1-\alpha)\right]D(q)+\beta\alpha U(q)\right\} \geq 0$. Fix any $n_b > 0$ and $z_b > 0$ such that $n_b + z_b = D(q^*)$ and

$$\beta\alpha\left[d(z_b)-C\left[q(z_b)\right]\right]+(\mu-1)z_b \leq \varepsilon_0.$$

Consider the first-best allocation $(q^*, z_b, 0, n_b, 0)$ and the following e-money mechanism $\mathcal{M}_E$:

$$T_s(z,n) = \beta\alpha\left[D(q^*)-C(q^*)-d(z_b)+C\left[q(z_b)\right]\right], \tag{C.3}$$

$$T_b(z,n) = \begin{cases} -T_s(z_n,n_n)+(\mu-1)n_b, \text{ if } z=z_b \text{ and } n=n_b \\ 0, \text{ otherwise} \end{cases}. \tag{C.4}$$

Then it is straightforward to verify that (C.3) implies (23) and that (C.4) implies (24) under the first-best allocation $(q^*, z_b, 0, n_b, 0)$ and $\mathcal{M}_E$ constructed above. So $(q^*, z_b, 0, n_b, 0)$ is incentive compatible for sellers under $\mathcal{M}_E$, and $\mathcal{M}_E$ is self-financed with limited participation. Finally, substituting $\mu d^* = z_b + n_b$, and (C.4) into $-\left[\mu-\beta(1-\alpha)\right]D(q^*)-$

$T_b(z_b, n_b) + \beta\alpha U(q^*) - \max_q \{-[\mu - \beta(1-\alpha)] D(q) + \beta\alpha U(q)\}$, we have

$$\begin{aligned}
& T_b(z_b, n_b) - [\mu - \beta(1-\alpha)] D(q^*) + \beta\alpha U(q^*) - \max_q \{-[\mu - \beta(1-\alpha)] D(q) + \beta\alpha U(q)\} \\
= & -[\mu - \beta(1-\alpha)] D(q^*) + (\mu - 1)(z_b + n_b) + \beta\alpha [D(q^*) - C(q^*)] + \beta\alpha U(q^*) \\
& -\beta\alpha [d(z_b) - C[q(z_b)]] - (\mu - 1) z_b + \max_q \{-[\mu - \beta(1-\alpha)] D(q) + \beta\alpha U(q)\} \\
\geq & -(1-\beta) D(q^*) + \beta\alpha [U(q^*) - C(q^*)] - \max_q \{-[\mu - \beta(1-\alpha)] D(q) + \beta\alpha U(q)\} - \varepsilon_0 \\
= & \varepsilon_0 - \varepsilon_0 = 0.
\end{aligned}$$

Therefore, (21) is satisfied given the first-best allocation $(q^*, z_b, 0, n_b, 0)$ and $\mathcal{M}_E$ constructed above. Thus $(q^*, z_b, 0, n_b, 0)$ is also incentive compatible for buyers under $\mathcal{M}_E$, and $\mathcal{M}_E$ implements the first best with limited participation.

D Proof of Proposition 7

Suppose that this is not the case, i.e., there does not exist any money mechanism but an e-money mechanism $\mathcal{M}_E = \{T_b(z,n), T_s(z,n)\}$ that implements some first-best allocation $(q^*, z_b, z_s, n_b, n_s)$ with some μ and $T_s(z_n, n_n) \leq 0$. Consider a first-best allocation (q^*, z_b', z_s') under a money mechanism $\mathcal{M} = \{T_b(z), T_s(z), \mu\}$, where $z_b' = z_b + n_b$, $z_s' = 0$, $T_s(z) = 0$ for all z, and

$$T_b(z) = \begin{cases} T_b(z_n, n_b) + A, \text{ if } z = z_b' \\ 0, \text{ otherwise,} \end{cases}$$

where

$$A \equiv (\mu - 1)(n_s + n_b + z_s + z_b) + T_s(z_n, n_n).$$

Notice that $A \geq 0$ due to the premise $T_s(z_n, n_n) \geq 0$. Then it is straightforward to verify that (13) is satisfied under the first-best allocation (q^*, z_b', z_s') with $\mathcal{M}$, since $z_s' = T_s(z) = 0$. Also, notice that

$$\begin{aligned}
& T_b(z_b) - [\mu - \beta(1-\alpha)] d^* + \beta\alpha U(q^*) \\
= & T_b(z_n, n_b) - [\mu - \beta(1-\alpha)] D(q^*) + \beta\alpha U(q^*) - A \\
\geq & \max_q \{-[\mu - \beta(1-\alpha)] D(q) + \beta\alpha U(q)\} - A,
\end{aligned}$$

where the last inequality comes from the fact that $(q^*, z_b, z_s, n_b, n_s)$ is incentive compatible to buyers under $\mathcal{M}_E$. So (11) is satisfied under the first-best allocation (q^*, z_b', z_s') with $\mathcal{M}$. Finally, it is straightforward to verify that (14) is satisfied under the first-best allocation (q^*, z_b', z_s') with $\mathcal{M}$. Thus (q^*, z_b', z_s') is incentive compatible to buyers and sellers under $\mathcal{M}$, and $\mathcal{M}$ is self-financed. This leads to a contradiction, since there exists a money mechanism $\mathcal{M}$ that implements the first best. Given $\mu \geq 1$, the result that $T_s(z_n, n_n) < 0$ implies $T_b(z_n, n_n) > 0$ from the e-money issuer's budget (24).

E Implementation by Indirect Mechanisms with limited participation: Membership-Reward-Deposit E-money

Proposition 4 states that when $\theta \geq \Theta(\theta, \mu)$, there exists a non-empty set of direct mechanisms that implements the first-best allocation with limited participation. We are also interested in constructing some simple indirect mechanisms that can implement the first-best allocation with limited participation. Consider that the e-money issuer charges sellers a fixed membership fee B_s to use e-money in the coming DM. To use e-money, buyers have to maintain a deposit of at least z_b units of real money balances, for a return in terms of a fixed reward $-B_b$ units of real e-money balances, and a proportional reward at a rate R to load e-money in the CM. The deposit can be used in the DM. Thus, a membership-reward-deposit mechanism is indexed by $\mathcal{M}_M \equiv \{B_s, B_b, R, z_b\}$. The optimization problem of a buyer in the CM under a membership-reward-deposit mechanism can be formulated as

$$\max_{z', n', q, q', \varepsilon_b \in \{0,1\}} \{e[-z' - n' + \beta U(q)] + (1-e)[-\mu D(q') + \beta U(q')]\}, \text{ s.t.} \qquad \text{(E.1)}$$

$$\begin{aligned} D(q) &= \frac{z' - B_b}{\mu} + \frac{1+R}{\mu} n', \\ z' &\geq z_b. \end{aligned}$$

The optimization problem of a seller in the CM under a membership-reward-deposit mechanism can be formulated as

$$\max_{e_s \in \{0,1\}} \left\{ e\left[-B_s + \beta\left[d - C\left(q\right)\right]\right] + \left(1-e\right)\beta\left[d\left(\frac{z_b}{\mu}\right) - C\left[q\left(\frac{z_b}{\mu}\right)\right]\right]\right\}. \tag{E.2}$$

Definition 16 *A membership-reward-deposit mechanism* $\mathcal{M}_M = \{B_s, B_b, R, z_b\}$ *is self-financed under the allocation* $(q, d, z_b, 0, n_b, 0)$ *if*

$$0 = B_s + B_b - \frac{R}{1+R}\left(n_b + B_b\right) + \left(1 - \mu^{-1}\right) n_b. \tag{E.3}$$

Definition 17 *A membership-reward-deposit mechanism* $\mathcal{M}_M$ *implements the first best if*

a. $z' = z_b$, $n' = \frac{n_b + B_b}{1+R}$, $q = q^*$ *and* $e_b = 1$ *solves (E.1);*

b. $e_s = 1$ *solves (E.2);*

c. $\mathcal{M}_M$ *is self-financed under the first-best allocation* $(q^*, d^*, z_b, 0, n_b, 0)$.

Define $\varepsilon_0 \equiv -\left(1-\beta\right)d^* + \beta\left[U\left(q^*\right) - C\left(q^*\right)\right] - \max_q\left\{-\mu D\left(q\right) + \beta U\left(q\right)\right\}$, where $\varepsilon_0 \geq 0$ if and only if $\theta \geq \Theta\left(\theta, \mu\right)$. The following proposition characterizes the set of membership-reward-deposit mechanisms that implements the first best.

Proposition 13 *Suppose that* $\theta \geq \Theta\left(\theta, \mu\right)$ *and that* $\mu \geq 1$. *There exists a membership-reward-deposit mechanism that implements the first best, which is constructed as follows:*

a. $R = \frac{\mu}{\beta} - 1$;

b. any $n_b > 0$ *and* $z_b > 0$ *such that* $n_b + z_b = \mu d^*$ *and* $\beta\left[d\left(\frac{z_b}{\mu}\right) - C\left[q\left(\frac{z_b}{\mu}\right)\right]\right] + \left(1 - \frac{1}{\mu}\right) z_b \leq \varepsilon_0$;

c. $B_s \in \left[0, \beta\left[d^* - C\left(q^*\right)\right] + \left(1 - \frac{1}{\mu}\right) z_b - \varepsilon_0\right]$; *and*

d. $B_b = -\mu B_s/\beta - \left(1 - \beta^{-1}\right) n_b$.

Proof. First, notice that

$$\begin{aligned}
& \beta\left[d^* - C\left(q^*\right)\right] + \left(1 - \frac{1}{\mu}\right) z_b - \varepsilon_0 \\
\geq\ & \beta\left[d^* - C\left(q^*\right)\right] + \left(1 - \frac{1}{\mu}\right) z_b - \beta\left[d\left(\frac{z_b}{\mu}\right) - C\left[q\left(\frac{z_b}{\mu}\right)\right]\right] - \left(1 - \frac{1}{\mu}\right) z_b \\
=\ & \beta\left[d^* - C\left(q^*\right) - d\left(\frac{z_b}{\mu}\right) + C\left[q\left(\frac{z_b}{\mu}\right)\right]\right] \geq 0,
\end{aligned}$$

so the set $\left[0, \beta\left[d^* - C\left(q^*\right)\right] + \left(1 - \frac{1}{\mu}\right) z_b - \varepsilon_0\right]$ in (c) is well-defined. Combining (b) and (c), we have

$$-B_s + \beta\left[d^* - C\left(q^*\right)\right] \geq \beta\left[d\left(\frac{z_b}{\mu}\right) - C\left[q\left(\frac{z_b}{\mu}\right)\right]\right],$$

so $e_s = 1$ satisfies (E.2). Substituting $D(q) = \frac{z' - B_b}{\mu} + \frac{1+R}{\mu} n'$ into (E.1), we have (E.1) equivalent to

$$\max_{\substack{z' \geq z_b, q, \\ q', e_b \in \{0,1\}}} \left\{ e\left[-\left(1 - \frac{\beta}{\mu}\right) z' - \frac{\beta}{\mu} B_b + \beta\left[U(q) - D(q)\right]\right] + (1 - e)\left[-\mu D\left(q'\right) + \beta U\left(q'\right)\right] \right\}, \tag{E.4}$$

where $n' = \frac{n_b + B_b}{1+R}$, $z' = z_b$ and $q = q^*$ solve the above. Substituting (c) and (d) into $-\left(1 - \frac{\beta}{\mu}\right) z_b - \frac{\beta}{\mu} B_b + \beta\left[U\left(q^*\right) - D\left(q^*\right)\right]$, we have

$$\begin{aligned} & -\left(1 - \frac{\beta}{\mu}\right) z_b - \frac{\beta}{\mu} B_b + \beta\left[U\left(q^*\right) - D\left(q^*\right)\right] \\ \geq & -\left(\frac{1-\beta}{\mu}\right)\left(z_b + n_b\right) + \beta\left[U\left(q^*\right) - C\left(q^*\right)\right] - \varepsilon_0 \\ = & \max_q \left\{-\mu D(q) + \beta U(q)\right\}. \end{aligned}$$

So $e_b = 1$ satisfies (E.1). Finally, the construction of B_b from (d) implies that (E.3) is satisfied. Thus, any membership-reward-deposit mechanism that satisfies (a) to (d) implements the first best. ■

F Proof of Proposition 8

In the interest of brevity, we show only the later part that if $\theta < \bar{\theta}$, then $\mathcal{M}_L$ implements the first best if and only if $\mu \geq \Phi(\theta)$. The proof of the rest of Proposition 8 just follows the proof of Proposition 4. Define

$$\eta \equiv T_s\left(z_s, n_s\right) - (\mu - \beta)\left(z_s + n_s\right).$$

The fact that $(q^*, z_b, z_s, n_b, n_s)$ is incentive compatible for sellers and buyers implies that $\eta \geq 0$ and $D\left(q^*\right) + \theta\Delta - \Delta_b = z_b + n_b$. Substituting (38) and $D\left(q^*\right) + \theta\Delta - \Delta_b = z_b + n_b$

into (37), we have

$$\begin{aligned} & T_b(z_b, n_b) - [\mu - \beta(1-\alpha)][D(q^*) + \theta\Delta] + \beta\alpha U(q^*) \qquad \text{(F.1)} \\ = \; & -A - (1-\beta)U(q^*) + [1 - \beta(1-\alpha)]\theta[U(q^*) - C(q^*)] \\ & + [\alpha - \theta[1 - \beta(1-\alpha)]]\Delta, \end{aligned}$$

where

$$A = \eta + \left(1 - \frac{1}{\mu}\right)\Delta_b + (\mu - 1)z_b + (\mu - \beta)z_s + (1-\beta)n_s \geq 0.$$

To show the "if" part, notice that there exists $\mathcal{M}_L$ such that $A = 0$ and $\Delta = U(q^*) - C(q^*)$. Obviously, the corresponding $(q^*, z_b, z_s, n_b, n_s)$ with $A = 0$ is incentive compatible for sellers under $\mathcal{M}_L$. Also, the right-hand side of (F.1) becomes

$$\begin{aligned} & -(1-\beta)U(q^*) + [1 - \beta(1-\alpha)]\theta[U(q^*) - C(q^*)] \\ & + [\alpha - \theta[1 - \beta(1-\alpha)]][U(q^*) - C(q^*)], \\ = \; & (\beta + \alpha - 1)U(q^*) - \alpha C(q^*), \\ \geq \; & \max_q \{\beta\alpha U(q) - [\mu - \beta(1-\alpha)]D(q)\}, \end{aligned}$$

where the last line uses the premise that $\mu \geq \Phi(\theta)$. Thus $(q^*, z_b, z_s, n_b, n_s)$ is incentive compatible for buyers under $\mathcal{M}_L$, and hence implements the first best. Notice that the above part is true whether or not the premise $\theta < \overline{\theta}$ is true, since the proof does not depend on the condition $\theta < \overline{\theta}$.

To show the "only if" part, suppose that there exists $\mathcal{M}_L$ implementing the first best for some μ. Then, (F.1) becomes

$$\begin{aligned} & \max_q \{\beta\alpha U(q) - [\mu - \beta(1-\alpha)]D(q)\} \\ \leq \; & -(1-\beta)U(q^*) + [1 - \beta(1-\alpha)]\theta[U(q^*) - C(q^*)] \\ & + [\alpha - \theta[1 - \beta(1-\alpha)]]\Delta, \\ = \; & (\theta - \overline{\theta})[1 - \beta(1-\alpha)][U(q^*) - C(q^*)] + [\alpha - \theta[1 - \beta(1-\alpha)]]\Delta. \end{aligned}$$

Under the premise $\theta < \overline{\theta}$, the first term on the last line is negative; thus, the second term must be positive in order to not be less than the non-negative first line. Thus, we must have $\alpha - \theta[1 - \beta(1-\alpha)] > 0$. Since $\Delta \leq [U(q^*) - C(q^*)]$, then the last line

is less than $(\beta+\alpha-1)U(q^*)-\alpha C(q^*)$. Thus we have reached $\mu \geq \Phi(\theta)$.

G Proof of Proposition 9

To prove (a), suppose that $\mathcal{M}_E$ implements the first best. Then from the proof of Proposition 4 it is necessary to have

$$\begin{aligned}
& \max_q \{-[\mu-\beta(1-\alpha)]D(q)+\beta\alpha U(q)\} \\
\leq\ & -(1-\beta)D(q^*)+\beta\alpha[U(q^*)-C(q^*)], \\
=\ & -(1-\beta)[(1-\theta)U(q^*)+\theta C(q^*)]+\beta\alpha[U(q^*)-C(q^*)] \\
& -(\beta+\alpha-1)U(q^*)+\alpha C(q^*)+(\beta+\alpha-1)U(q^*)-\alpha C(q^*), \\
=\ & -[[1-\beta\alpha-(1-\beta)\theta]+(\beta+\alpha-1)][U(q^*)-C(q^*)]-\beta C(q^*) \\
& +(\beta+\alpha-1)U(q^*)-\alpha C(q^*), \\
\leq\ & -\beta U(q^*)+(\beta+\alpha-1)U(q^*)-\alpha C(q^*), \\
\leq\ & (\beta+\alpha-1)U(q^*)-\alpha C(q^*),
\end{aligned}$$

where the second-last inequality has used the premise that $\theta \leq \alpha$. Thus the last line implies that $\mu \geq \Phi(\theta)$. Then from the proof of Proposition 8 there exists an e-money mechanism $\mathcal{M}_L$ that implements the first best for the given μ.

To prove (b), suppose that $\mathcal{M}_L$ implements the first best. Then from the proof of Proposition 8 we have

$$\begin{aligned}
& \max_q \{-[\mu-\beta(1-\alpha)]D(q)+\beta\alpha U(q)\} \\
\leq\ & (\theta-\overline{\theta})[1-\beta(1-\alpha)][U(q^*)-C(q^*)]+[\alpha-\theta[1-\beta(1-\alpha)]]\Delta, \\
\leq\ & (\theta-\overline{\theta})[1-\beta(1-\alpha)][U(q^*)-C(q^*)]+\beta\alpha(1-\theta)[U(q^*)-C(q^*)], \\
=\ & [\beta\alpha(1-\theta)+[1-\beta(1-\alpha)](\theta-\overline{\theta})][U(q^*)-C(q^*)],
\end{aligned}$$

where the last inequality uses the fact that $\Delta \in [0, U(q^*)-C(q^*)]$ and the fact that $\theta > \alpha$ implies $\beta\alpha(1-\theta) > \alpha-\theta[1-\beta(1-\alpha)]$. Thus, we have $\mu > \Theta(\theta)$. Then by Proposition 4 there exists $\mathcal{M}_E$ implementing the first best for the given μ as well.

H Proof of Proposition 11

In the interest of brevity, we show only (a). Suppose that this is not the case, i.e., there exists an e-money mechanism $\mathcal{M}_L = \{\Delta_b, \Delta_s, T_b(z,n), T_s(z,n)\}$ that implements some first-best allocation $(q^*, z_b, z_s, n_b, n_s)$ with limited transferability under $\Delta = 0$ and some μ, but there does not exist any e-money mechanism $\mathcal{M}_E = \{T_b(z,n), T_s(z,n)\}$ that implements the first-best allocation with limited participation under the same μ. Given $\Delta = 0$, consider a money mechanism $\mathcal{M}_L = \{T_b'(z), T_s'(z), \mu\}$ where

$$T_s'(z) = \begin{cases} T_s(z_s, n_s), \text{ if } z = z_s + n_s \\ 0, \text{ otherwise} \end{cases}$$

$$T_b'(z) = T_b(z,n) + (\mu - 1)(z_b + z_s).$$

Consider a first-best allocation (q^*, z_b', z_s') where $z_b' = z_b + n_b$ and $z_s' = z_s + n_s$. Then it is straightforward to verify that (13) is satisfied under the first-best allocation (q^*, z_b', z_s') with $\mathcal{M}$, since (13) and (37) are the same. Also, notice that

$$\begin{aligned} & T_b(z_b) - [\mu - \beta(1-\alpha)] D(q^*) + \beta\alpha U(q^*) \\ = \; & T_b(z_b, n_b) - [\mu - \beta(1-\alpha)] D(q^*) + \beta\alpha U(q^*) + \left(1 - \mu^{-1}\right)(z_b + z_s) \\ \geq \; & \max_q \{ - [\mu - \beta(1-\alpha)] D(q) + \beta\alpha U(q) \}, \end{aligned}$$

where the last inequality comes from the fact that $(q^*, z_b, z_s, n_b, n_s)$ is incentive compatible to buyers under $\mathcal{M}_L$. So (11) is satisfied under the first-best allocation (q^*, z_b', z_s') with $\mathcal{M}$. Finally, it is straightforward to verify that (14) is satisfied under the first-best allocation (q^*, z_b', z_s') with $\mathcal{M}$. Thus (q^*, z_b', z_s') is incentive compatible to buyers and sellers under $\mathcal{M}$, and $\mathcal{M}$ is self-financed. This contradicts Proposition 5, since there exists a money mechanism $\mathcal{M}$ that implements the first best but there does not exist any e-money mechanism $\mathcal{M}_E = \{T_b(z,n), T_s(z,n)\}$ that implements the first-best allocation with limited participation under the same μ. Therefore, we establish $\Delta > 0$. Finally, notice that (9) is satisfied only if $T_b(z_n, n_n) > 0$. Thus, we prove Proposition 11.

I Implementation by Indirect Mechanisms with Limited Transferability: Interchange-Reward-Deposit E-money

Proposition 8 states that when $\mu \geq \Phi(\theta)$, there exists a non-empty set of direct mechanisms that implements the first-best allocation with limited transferability. We are also interested in constructing some simple indirect mechanisms that can implement the first-best allocation with limited transferability. Consider that to use e-money, buyers have to maintain a deposit of at least z units of real money balances, for a return in terms of a fixed reward of $-B$ units of real e-money balances, and a proportional reward at a rate R to load e-money in the CM. The deposit can be used in the DM. To receive any positive amount of e-money in the DM, the payee is charged a fixed interchange fee of Δ units of real e-money balances from the e-money received. Thus, an interchange-reward-deposit mechanism is indexed by $\mathcal{M}_I \equiv \{\Delta, B, R, z\}$. The optimization problem of a buyer in the CM under an interchange-reward-deposit mechanism can be formulated as

$$\max_{z', n', q, q', e_b \in \{0,1\}} \{e\left[-z' - n' + \beta U(q)\right] + (1-e)\left[-\mu D(q') + \beta U(q')\right]\}, \text{ s.t.} \tag{I.1}$$

$$\begin{aligned} D(q) + \theta\Delta &= \frac{z' - B}{\mu} + \frac{1+R}{\mu} n', \\ z' &\geq z. \end{aligned}$$

Definition 18 *An interchange-reward-deposit mechanism $\mathcal{M}_I \equiv \{\Delta, B, R, z\}$ is self-financed under the allocation (q, d, z, n) if*

$$0 = \Delta + B - \frac{R}{1+R}(n+B) + \left(1 - \mu^{-1}\right) z. \tag{I.2}$$

Definition 19 *An interchange reward deposit mechanism $\mathcal{M}_I$ implements the first best if*

a. $z' = z$, $n' = \frac{n+B_b}{1+R}$, $q = q^*$ and $e_b = 1$ solves (I.1);

b. $\mathcal{M}_I$ is self financed under the first best allocation (q^*, d^*, z, n).

Define $\varepsilon_0 \equiv \beta U(q^*) - C(q^*) - \max_q \{-\mu D(q) + \beta U(q)\}$, where $\varepsilon_0 \geq 0$ if and only if $\mu \geq \Phi(\theta)$. The following proposition characterizes the set of interchange-reward-deposit mechanisms that implements the first best.

Proposition 14 *Suppose that $\mu \geq \Phi(\theta)$. There exists an interchange-reward-deposit mechanism that implements the first best, which is constructed as follows:*

a. $R = \frac{\mu}{\beta} - 1$;

b. any $\Delta \in [0, U(q^*) - C(q^*)]$, $n > 0$ *and* $z > 0$ *such that* $n + z = \mu(d^* + \theta\Delta)$ *and*

$$\left(1 - \frac{1}{\mu}\right) z + (1 - \theta)\left[U(q^*) - C(q^*) - \Delta\right] \leq \varepsilon_0;$$

c. $B = -(1 + R)\Delta - \left(1 - \beta^{-1}\right) n$.

Proof. Substituting $D(q) = \frac{z' - B}{\mu} + \frac{1+R}{\mu} n'$ into (I.1), we have (I.1) equivalent to

$$\max_{\substack{z' \geq z, q, \\ q', e_b \in \{0,1\}}} \left\{ e\left[-\left(1 - \frac{\beta}{\mu}\right) z' - \frac{\beta}{\mu} B + \beta\left[U(q) - D(q)\right]\right] + (1 - e)\left[-\mu D(q') + \beta U(q')\right]\right\}.$$

Then $n' = \frac{n+B}{1+R}$, $z' = z$ and $q = q^*$ solve the above. Together with (c), we have

$$\begin{aligned}
& -z' - n' + \beta U(q^*) \\
= \; & -z - \frac{n + B}{1 + R} + \beta U(q^*) \\
= \; & -\left(1 - \frac{1}{\mu}\right) z - (1 - \theta)\left[U(q) - C(q^*) - \Delta\right] + \beta U(q) - C(q^*) \\
\geq \; & -\varepsilon_0 + \beta U(q) - C(q^*) \\
= \; & \max_q \left\{-\mu D(q) + \beta U(q)\right\}.
\end{aligned}$$

So $e_b = 1$ satisfies (I.1). Finally, the construction of B from (c) implies that (I.2) is satisfied. Thus, any interchange-reward-deposit mechanism satisfying (a) to (c) implements the first best. ■

J Competitive Pricing

In this section we show that all the results under proportional bargaining extend to the environment with competitive pricing. In the DM, agents take a competitive price p as given. The seller's DM problem is given by

$$\max_{q} \{-C(q) + pq\}. \tag{J.1}$$

Thus the equilibrium price is given by

$$p = C'(q). \tag{J.2}$$

We can rewrite the buyer's optimization problem in the CM as

$$\max_{q} \{-[\mu - \beta(1-\alpha)]pq + \beta\alpha U(q)\}. \tag{J.3}$$

The following proposition characterizes the equilibrium.

Proposition 15 *Under competitive pricing, a monetary equilibrium exists iff $\mu \geq \beta$. If $\mu > \beta$, then $q < q^*$; if $\mu \to \beta$, then $q = q^*$.*

Proof. A monetary equilibrium exists if $q \in (0, \infty)$. The first-order condition with respect to q evaluated at the equilibrium price $p = C'(q)$ is then

$$-[\mu - \beta(1-\alpha)]C'(q) + \beta\alpha U'(q) = 0.$$

Since $q \leq q^*$ and hence $C'(q) \leq U'(q)$, the above has an interior solution of $q \in (0, \infty)$ iff $\mu \geq \beta$. Notice that q is strictly decreasing in μ. If $\mu = \beta$, we have $C'(q) = U'(q)$ and hence $q = q^*$. ■

J.1 Optimal Money Mechanism

It is straightforward to verify that, under competitive pricing, the buyer's CM problem under a money mechanism $\mathcal{M}$ is given by

$$\max_{q,\hat{z},e_b \in \{0,1\}} \{-e_b T_b(\hat{z}) - [\mu - \beta(1-\alpha)]pq + \beta\alpha U(q)\}, \text{ s.t.} \tag{J.4}$$

$$\widehat{z} \leq \mu p q. \tag{J.5}$$

To induce buyers to participate in the mechanism (i.e. $e_b = 1$), it is necessary to have an incentive-compatible allocation (q, d, z_b, z_s) satisfying

$$-T_b(z_b) - [\mu - \beta(1-\alpha)] C'(q) q + \beta\alpha U(q) \geq \max_{q'} \{ - [\mu - \beta(1-\alpha)] C'(q) q' + \beta\alpha U(q') \}. \tag{J.6}$$

Here, the LHS captures the payoff for participating in the mechanism at the equilibrium price $p = C'(q)$ and $d = pq$, and the RHS captures the payoff for skipping it.

Similarly, using the linearity of $W_s(z)$, and ignoring the constant terms, one can reformulate the seller's problem in the CM as

$$\max_{z', \widehat{z}, e_s \in \{0,1\}} \left\{ -z' - e_s T_s(\widehat{z}) + \frac{\beta}{\mu} z' \right\}, \text{ s.t. } \widehat{z} \leq z'. \tag{J.7}$$

Here, a seller not participating in the mechanism has no reason to bring money and thus the additional payoff is zero. A seller who decides to participate and intends to have a post-transfer balance of z needs to bring $z + T_s(\widehat{z})$ from the CM, and report $\widehat{z} \leq z'$, where the balance will have a continuation value $\beta z'/\mu$.

Notice that the value after choosing $e_s = 0$ is zero. So to induce sellers to participate in the mechanism, it is necessary to have an incentive-compatible allocation (q, d, z_b, z_s) satisfying

$$-z_s - T_s(z_s) + \frac{\beta}{\mu} z_s \geq 0. \tag{J.8}$$

Here, the LHS captures the payoff for participating in the mechanism, and the RHS captures the payoff for skipping it.

With abuse of notation, define the share of buyer surplus at the first best as

$$\theta \equiv \frac{U(q^*) - C'(q^*) q^*}{U(q^*) - C(q^*)}. \tag{J.9}$$

The following proposition characterizes the implementability of the first-best allocation under an optimally designed money mechanism.

Proposition 16 *Under competitive pricing, there exists a money mechanism $\mathcal{M}$ that implements the first best if and only if $\theta \geq \overline{\theta}$.*

Proof. We sketch the proof as follows. First, we show that if $\theta < \overline{\theta}$ then there does

not exist any money mechanism $\mathcal{M}$ that implements the first best. Suppose that this is not the case. Denote (q^*, d^*, z_b, z_s) as the first-best allocation implemented with $p = C'(q^*)$. Since the equilibrium exists as the first-best allocation, we must have $\mu \geq \beta$. Denote $\eta \equiv -\left(1 - \frac{\beta}{\mu}\right) z_s - T_s(z_s)$. Since (q^*, d^*, z_b, z_s) is incentive compatible for sellers, we have $\eta \geq 0$. The fact that (q^*, d^*, z_b, z_s) is incentive compatible for buyers implies that $\mu d^* = z_b$. Substituting $\mu d^* = z_b$ into the definition of η, we have

$$-T_b(z_b) - \mu d^* = -\eta - \frac{1-\beta}{\mu} z_s - d^*. \tag{J.10}$$

Since (q^*, d^*, z_b, z_s) is also incentive compatible for buyers, we have

$$\begin{aligned} & \max_{q'} \left\{ -[\mu - \beta(1-\alpha)] pq' + \beta\alpha U(q') \right\}, \\ \leq\ & -T_b(z_b) - [\mu - \beta(1-\alpha)] d^* + \beta\alpha U(q^*), \\ =\ & \underbrace{-\eta - \frac{1-\beta}{\mu} z_s}_{\leq 0} + \left(\theta - \overline{\theta}\right) [1 - \beta(1-\alpha)] [U(q^*) - C(q^*)] \leq 0, \end{aligned}$$

where we have used the fact that

$$\begin{aligned} & \beta\alpha U(q^*) - [1 - \beta(1-\alpha)] d^*, \\ =\ & \beta\alpha U(q^*) - [1 - \beta(1-\alpha)] [(1-\theta) U(q^*) + \theta C(q^*)], \\ =\ & \left(\theta - \overline{\theta}\right) [1 - \beta(1-\alpha)] [U(q^*) - C(q^*)] < 0. \end{aligned}$$

Since we have $\max_{q'} \{ -[\mu - \beta(1-\alpha)] pq' + \beta\alpha U(q') \} \geq 0$, there is a contradiction.

On the other hand, if $\theta \geq \overline{\theta}$, we can construct a money mechanism $\mathcal{M}$ that implements the first best. Consider the following money mechanism: $T_s(z) = 0$ for all z, $\mu = \overline{\mu}$ and

$$T_b(z) = \begin{cases} (1 - \overline{\mu}) d^*, \text{ if } z = \overline{\mu} d^* \\ 0, \text{ otherwise.} \end{cases}$$

■

Proposition 17 *Under competitive pricing, if a money mechanism $\mathcal{M} = \{T_b(z), T_s(z), \mu\}$ implements the first best, then $\mu > 1$.*

Proof. Suppose that there exists a mechanism $\mathcal{M} = \{T_b(z), T_s(z), \mu\}$ that implements the first best with $\mu \leq 1$. Then from the proof of the previous proposition, we

have

$$\begin{aligned} & -[1-\beta(1-\alpha)]C'(q^*)q^* + \beta\alpha U(q^*) \\ \geq\ & -T_b(z_b) - [\mu - \beta(1-\alpha)]d^* + \beta\alpha U(q^*), \\ \geq\ & \max_{q'}\{-[\mu-\beta(1-\alpha)]C'(q^*)q' + \beta\alpha U(q')\}, \\ \geq\ & \max_{q'}\{-[1-\beta(1-\alpha)]pq' + \beta\alpha U(q')\}, \end{aligned}$$

which is a contradiction, since q^* is the maximizer to the last line only if $\beta = 1$. ■

J.2 Electronic Money with Limited Participation

In this section we consider the e-money mechanism $\mathcal{M}_E$ with limited participation without money, but with the e-money growth rate exogenously given by μ. It is straightforward to verify that under competitive pricing with an e-money mechanism $\mathcal{M}_E$, the buyer's problem in the CM under $e_s = 1$ is

$$\max_{e_b\in\{0,1\},\widehat{n},q}\{-[\mu-\beta(1-\alpha)]pq - e_b T_b(\widehat{n}) + \beta\alpha U(q)\}, \text{ s.t.} \tag{J.11}$$

$$\widehat{n} \leq \mu pq.$$

To induce buyers to participate in the mechanism, it is necessary to have an incentive-compatible allocation (q, d, n_b, n_s) satisfying

$$-T_b(n_b) - [\mu-\beta(1-\alpha)]C'(q)q + \beta\alpha U(q) \geq \max_{q'}\{-[\mu-\beta(1-\alpha)]C'(q)q' + \beta\alpha U(q')\}. \tag{J.12}$$

Here, the LHS captures the payoff for joining the e-money mechanism at the equilibrium price $p = C'(q)$, and the RHS captures the payoff for skipping it.

Similarly, using the linearity of $W_s(a)$, and ignoring the constant terms, one can reformulate the seller's problem in the CM under $e_b = 1$ as

$$\max_{\substack{e_s\in\{0,1\},\\ \widehat{n},n',q}}\left\{-z' - n' - e_s T_s(\widehat{n}) + \beta\left(\frac{n'}{\mu}\right) + \beta\alpha e_s[pq - C(q)]\right\}, \text{ s.t.} \tag{J.13}$$

$$\widehat{n} \leq n'.$$

Again, a seller joining the e-money mechanism (i.e. $e_s = 1$) has to bring extra balances

to pay for the transfer $T_s(\widehat{n})$.

To induce sellers to participate in the mechanism, it is necessary to have an incentive-compatible allocation (q, d, n_b, n_s) satisfying

$$-T_s(n_s) - \left(1 - \frac{\beta}{\mu}\right) n_s + \beta\alpha \left[C'(q)\, q - C(q)\right] \geq 0, \tag{J.14}$$

where the LHS captures the payoff for participating in the e-money mechanism, and the RHS captures the payoff for skipping it.

Define Θ as the solution to

$$-(1-\beta)\, U(q^*) + [\beta\alpha + (1-\beta)\,\theta]\left[U(q^*) - C(q^*)\right] = \max_q \left\{\beta\alpha U(q) - [\Theta - \beta(1-\alpha)]\, C'(q^*)\, q\right\},$$

and set $\Theta = \infty$ if a solution $\Theta \geq \beta$ does not exist. The following proposition establishes the condition under which the first best can be achieved by an optimal e-money mechanism with limited participation.

Proposition 18 *Under competitive pricing, there exists an e-money mechanism $\mathcal{M}_E$ that implements the first best with limited participation if and only if $\mu \geq \Theta$.*

Proof. First, we want to show that if $\mu < \Theta$, then there does not exist an e-money mechanism $\mathcal{M}_E$ that implements the first best with limited participation. Suppose that this is not the case, then there exists an e-money mechanism $\mathcal{M}_E$ that implements a first best (q^*, d^*, n_b, n_s). Define

$$\eta \equiv -\left(1 - \frac{\beta}{\mu}\right) n_s - T_s(n_s) + \beta\alpha\left[d^* - C(q^*)\right].$$

The fact that (q^*, d^*, n_b, n_s) is incentive compatible for sellers and buyers implies that $\eta \geq 0$ and $\mu d^* = n_b$. Then we have

$$-[\mu - \beta(1-\alpha)]\, d^* - T_b(n_b) = -\eta - \left(\frac{1-\beta}{\mu}\right) n_s - \beta\alpha C(q^*) - (1-\beta)\, d^*. \tag{J.15}$$

Notice that the definition of Θ implies that $\mu < \Theta$ if and only if

$$\left[\beta\alpha(1-\theta) + [1 - \beta(1-\alpha)]\left(\theta - \overline{\theta}\right)\right]\left[U(q^*) - C(q^*)\right] < \max_q \left\{-[\mu - \beta(1-\alpha)]\, C'(q^*)\, q + \beta\alpha U(q)\right\}. \tag{J.16}$$

Since (q^*, d^*, n_b, n_s) is also incentive compatible for buyers, we have

$$\begin{aligned}
& \max_q \left\{ -\left[\mu - \beta(1-\alpha)\right] C'(q^*) q + \beta\alpha U(q) \right\} \\
\leq\ & -T_b(n_b) - \left[\mu - \beta(1-\alpha)\right] d^* + \beta\alpha U(q^*) \\
=\ & -\eta - \left(\frac{1-\beta}{\mu}\right) n_s - (1-\beta) d^* + \beta\alpha \left[U(q^*) - C(q^*)\right] \\
=\ & -\eta - \left(\frac{1-\beta}{\mu}\right) n_s + \left[\beta\alpha(1-\theta) + (\theta - \bar{\theta})\left[1 - \beta(1-\alpha)\right]\right] \left[U(q^*) - C(q^*)\right] \\
<\ & -\eta - \left(\frac{1-\beta}{\mu}\right) n_s + \max_q \left\{ -\left[\mu - \beta(1-\alpha)\right] D(q) + \beta\alpha U(q) \right\}.
\end{aligned}$$

A contradiction.

On the other hand, if $\mu \geq \Theta$, we can construct an e-money mechanism $\mathcal{M}_E$ that implements the first best with limited participation. Since $\theta \geq \Theta$, we have $\varepsilon_0 \equiv -(1-\beta) d^* + \beta\alpha \left[U(q^*) - C(q^*)\right] - \max_q \left\{ -\left[\mu - \beta(1-\alpha)\right] C'(q^*) q + \beta\alpha U(q) \right\} \geq 0$. Consider the first-best allocation $(q^*, d^*, n_b, 0)$ and the following e-money mechanism $\mathcal{M}_E$:

$$T_s(n) = \beta\alpha \left[d^* - C(q^*)\right], \tag{J.17}$$

$$T_b(n) = \begin{cases} -T_s(n_s) - \left(1 - \frac{1}{\mu}\right) n_b, \text{ if } z = z_b \text{ and } n = n_b \\ 0, \text{ otherwise} \end{cases}. \tag{J.18}$$

Then it is straightforward to verify that $(q^*, d^*, n_b, 0)$ is incentive compatible for sellers under $\mathcal{M}_E$, and that $\mathcal{M}_E$ is self-financed with limited participation. Finally, substituting $\mu d^* = n_b$ into $-\left[\mu - \beta(1-\alpha)\right] d^* - T_b(n_b) + \beta\alpha U(q^*) - \max_q \left\{ -\left[\mu - \beta(1-\alpha)\right] C'(q^*) q + \beta\alpha U(q) \right\}$, we have

$$\begin{aligned}
& -\left[\mu - \beta(1-\alpha)\right] d^* - T_b(n_b) + \beta\alpha U(q^*) - \max_q \left\{ -\left[\mu - \beta(1-\alpha)\right] C'(q^*) q + \beta\alpha U(q) \right\} \\
=\ & -\left[\mu - \beta(1-\alpha)\right] d^* + \left(1 - \frac{1}{\mu}\right) n_b + \beta\alpha \left[d^* - C(q^*)\right] + \beta\alpha U(q^*) \\
& + \max_q \left\{ -\left[\mu - \beta(1-\alpha)\right] C'(q^*) q + \beta\alpha U(q) \right\} \\
\geq\ & -(1-\beta) d^* + \beta\alpha \left[U(q^*) - C(q^*)\right] - \max_q \left\{ -\left[\mu - \beta(1-\alpha)\right] C'(q^*) q + \beta\alpha U(q) \right\} - \varepsilon_0 \\
=\ & \varepsilon_0 - \varepsilon_0 = 0.
\end{aligned}$$

Thus $(q^*, d^*, n_b, 0)$ is also incentive compatible for buyers under $\mathcal{M}_E$, and $\mathcal{M}_E$ implements the first best with limited participation. ■

Proposition 19 *Under competitive pricing, if there exists a money mechanism $\mathcal{M}$ that implements the first best with μ, then there also exists an e-money mechanism $\mathcal{M}_E$ that implements the first best with limited participation under the same μ.*

Proof. Since $\mathcal{M}$ implements the first best, from the proof before it is necessary to have

$$\begin{aligned} 0 &\leq -[1-\beta(1-\alpha)]\,d^* + \beta\alpha U(q^*) - \max_q \{-[\mu-\beta(1-\alpha)]\,C'(q^*)\,q + \beta\alpha U(q)\} \\ &= -(1-\beta)\,d^* + \beta\alpha\theta\,[U(q^*) - C(q^*)] - \max_q \{-[\mu-\beta(1-\alpha)]\,C'(q^*)\,q + \beta\alpha U(q)\} \\ &\leq -(1-\beta)\,d^* + \beta\alpha\,[U(q^*) - C(q^*)] - \max_q \{-[\mu-\beta(1-\alpha)]\,C'(q^*)\,q + \beta\alpha U(q)\} \\ &= (1-\beta)\,[1-\beta(1-\alpha)]\,[U(q^*) - C(q^*)]\,(\theta-\Theta). \end{aligned}$$

Thus we have $\theta \geq \Theta$, and therefore from the previous proposition there exists an e-money mechanism $\mathcal{M}_E$ that implements the first best. ■

Proposition 20 *If $\theta \in [\underline{\theta}, \bar{\theta})$, then under competitive pricing, first best allocation*

(i) cannot be implemented by any money mechanism;

(ii) can be implemented by an e-money mechanism with limited participation under some μ.

Proof. Omitted here. ■

Proposition 21 *Given μ, under competitive pricing if there exists an e-money mechanism with limited participation $\mathcal{M}_E = \{T_b(n), T_s(n)\}$, but not any money mechanism $\mathcal{M} = \{T_b(z), T_s(z), \mu\}$, that implements the first best, then $T_s(n) > 0$ and $T_b(n) < 0$.*

Proof. Omitted here. ■

J.3 Electronic Money with Limited Transferability

In this section we consider the e-money mechanism $\mathcal{M}_L$ with limited transferability, again without money but with the e-money growth rate exogenously given by μ. As before, to induce buyers to participate in the mechanism, it is necessary to have an incentive-compatible allocation (q, d_z, d_n, n_b, n_s), where $e_b = 1$ and $d_n > 0$, satisfying

$$-T_b(n_b) - [\mu-\beta(1-\alpha)]\,C'(q)\,q + \beta\alpha U(q) \geq \max_{q'} \{-[\mu-\beta(1-\alpha)]\,C'(q)\,q' + \beta\alpha U(q')\}. \tag{J.19}$$

Similarly, to induce sellers to participate in the mechanism, it is necessary to have an incentive-compatible allocation (q, d_z, d_n, n_b, n_s) such that, $e_s = 1$, satisfying

$$-\left(1-\frac{\beta}{\mu}\right) n_s - T_s(n_s) \geq 0. \tag{J.20}$$

As before, define Φ as the solution to

$$(\beta+\alpha-1)U(q^*) - \alpha C(q^*) = \max_q \left\{\beta\alpha U(q) - [\Phi - \beta(1-\alpha)] C'(q^*) q\right\},$$

and set $\Phi = \infty$ if a solution $\Phi \geq \beta$ does not exist. The following proposition establishes when the optimal e-money mechanism featuring limited transferability is efficient.

Proposition 22 *Under competitive pricing, there exists some μ and an e-money mechanism $\mathcal{M}_L$ that implements the first best with limited transferability if and only if either (a) $\theta \geq \overline{\theta}$ or (b) $(\beta+\alpha-1)U(q^*) > \alpha C(q^*)$. If (a) does not hold, then there exists $\mathcal{M}_L$ implementing the first best if and only if $\mu \geq \Phi$.*

Proof. In the interest of brevity, we show only the latter part that if $\theta < \overline{\theta}$, then $\mathcal{M}_L$ implements the first best if and only if $\mu \geq \Phi$. Define

$$\eta \equiv -\left(1-\frac{\beta}{\mu}\right) n_s - T_s(n_s).$$

The fact that (q^*, d^*, n_b, n_s) is incentive compatible for sellers and buyers implies that $\eta \geq 0$ and $\mu(d^* + \Delta_b) = n_b$. The issuer's budget is

$$0 = \alpha(\Delta_b + \Delta_s) + T_b(n_b) + T_s(n_s) + \left(1-\frac{1}{\mu}\right)(n_b + n_s). \tag{J.21}$$

Substituting $\mu(d^* + \Delta_b) = n_b$ and the issuer's budget, we have

$$\begin{aligned}
& -[\mu - \beta(1-\alpha)](d^* + \Delta_b) - T_b(n_b) + \beta\alpha U(q^*) \\
= \; & -[1-\beta(1-\alpha)](d^* + \Delta_b) + \alpha\Delta + T_s(n_s) + \left(1-\frac{1}{\mu}\right) n_s + \beta\alpha U(q^*) \\
= \; & -\eta - (1-\beta)U(q^*) + [1-\beta(1-\alpha)]\theta[U(q^*) - C(q^*)] \\
& + [\alpha - \theta[1-\beta(1-\alpha)]]\Delta.
\end{aligned}$$

To show the "if" part, notice that there exists $\mathcal{M}_L$ such that $\Delta = U(q^*) - C(q^*)$. Obviously, the corresponding $(q^*, d_z, d_n, n_b, n_s)$ with $A = 0$ is incentive compatible for

sellers under $\mathcal{M}_L$. Then we have

$$\begin{aligned} & -(1-\beta)U(q^*) + [1-\beta(1-\alpha)]\theta[U(q^*) - C(q^*)] \\ & + [\alpha - \theta[1-\beta(1-\alpha)]][U(q^*) - C(q^*)], \\ = \; & (\beta+\alpha-1)U(q^*) - \alpha C(q^*), \\ \geq \; & \max_q \{\beta\alpha U(q) - [\mu - \beta(1-\alpha)]C'(q^*)q\}, \end{aligned}$$

where the last line uses the premise that $\mu \geq \Phi(\theta)$. Thus $(q^*, d_z, d_n, z_b, z_s, n_b, n_s)$ is incentive compatible for buyers under $\mathcal{M}_L$, and hence implements the first best. Notice that the above part is true whether or not the premise $\theta < \overline{\theta}$ is true, since the proof does not depend on the condition $\theta < \overline{\theta}$.

To show the "only if" part, suppose there exists $\mathcal{M}_L$ implementing the first best for some μ. Then, we have

$$\begin{aligned} & \max_q \{\beta\alpha U(q) - [\mu - \beta(1-\alpha)]C'(q^*)q\} \\ \leq \; & -(1-\beta)U(q^*) + [1-\beta(1-\alpha)]\theta[U(q^*) - C(q^*)] \\ & + [\alpha - \theta[1-\beta(1-\alpha)]]\Delta, \\ = \; & (\theta - \overline{\theta})[1-\beta(1-\alpha)][U(q^*) - C(q^*)] + [\alpha - \theta[1-\beta(1-\alpha)]]\Delta. \end{aligned}$$

Under the premise $\theta < \overline{\theta}$, the first term on the last line is negative, and thus the second term must be positive in order to be not less than the non-negative first line. Thus, we must have $\alpha - \theta[1-\beta(1-\alpha)] > 0$. Since $\Delta \leq [U(q^*) - C(q^*)]$, the last line is less than $(\beta+\alpha-1)U(q^*) - \alpha C(q^*)$. Thus we have reached $\mu \geq \Phi$. ■

Proposition 23 *Under competitive pricing,*

(a) Suppose that $\theta \leq \alpha$. If there exists an e-money mechanism $\mathcal{M}_E$ that implements the first best with limited participation, then there also exists an e-money mechanism $\mathcal{M}_L$ that implements the first best with limited transferability under the same μ.

(b) Suppose that $\theta > \alpha$. If there exists an e-money mechanism $\mathcal{M}_L$ that implements the first best with limited transferability, then there also exists an e-money mechanism $\mathcal{M}_E$ that implements the first best with limited participation under the same μ.

Proof. To prove (a), suppose that $\mathcal{M}_E$ implements the first best. Then from the proof

of the previous proposition it is necessary to have

$$\begin{aligned}
&\max_{q}\left\{-\left[\mu-\beta\left(1-\alpha\right)\right]C'\left(q^{*}\right)q+\beta\alpha U\left(q\right)\right\}\\
\leq\ & -\left(1-\beta\right)d^{*}+\beta\alpha\left[U\left(q^{*}\right)-C\left(q^{*}\right)\right],\\
=\ & -\left(1-\beta\right)\left[\left(1-\theta\right)U\left(q^{*}\right)+\theta C\left(q^{*}\right)\right]+\beta\alpha\left[U\left(q^{*}\right)-C\left(q^{*}\right)\right]\\
& -\left(\beta+\alpha-1\right)U\left(q^{*}\right)+\alpha C\left(q^{*}\right)+\left(\beta+\alpha-1\right)U\left(q^{*}\right)-\alpha C\left(q^{*}\right),\\
=\ & -\left[\left[1-\beta\alpha-\left(1-\beta\right)\theta\right]+\left(\beta+\alpha-1\right)\right]\left[U\left(q^{*}\right)-C\left(q^{*}\right)\right]-\beta C\left(q^{*}\right)\\
& +\left(\beta+\alpha-1\right)U\left(q^{*}\right)-\alpha C\left(q^{*}\right),\\
\leq\ & -\beta U\left(q^{*}\right)+\left(\beta+\alpha-1\right)U\left(q^{*}\right)-\alpha C\left(q^{*}\right),\\
\leq\ & \left(\beta+\alpha-1\right)U\left(q^{*}\right)-\alpha C\left(q^{*}\right),
\end{aligned}$$

where the second-last inequality has used the premise that $\theta \leq \alpha$. Thus the last line implies that $\mu \geq \Phi$. Then from the proof of the previous proposition there exists an e-money mechanism $\mathcal{M}_L$ that implements the first best for the given μ.

To prove (b), suppose that $\mathcal{M}_L$ implements the first best. Then from the proof of the previous proposition we have

$$\begin{aligned}
&\max_{q}\left\{-\left[\mu-\beta\left(1-\alpha\right)\right]C'\left(q^{*}\right)q+\beta\alpha U\left(q\right)\right\}\\
\leq\ & \left(\theta-\overline{\theta}\right)\left[1-\beta\left(1-\alpha\right)\right]\left[U\left(q^{*}\right)-C\left(q^{*}\right)\right]+\left[\alpha-\theta\left[1-\beta\left(1-\alpha\right)\right]\right]\Delta,\\
\leq\ & \left(\theta-\overline{\theta}\right)\left[1-\beta\left(1-\alpha\right)\right]\left[U\left(q^{*}\right)-C\left(q^{*}\right)\right]+\beta\alpha\left(1-\theta\right)\left[U\left(q^{*}\right)-C\left(q^{*}\right)\right],\\
=\ & \left[\beta\alpha\left(1-\theta\right)+\left[1-\beta\left(1-\alpha\right)\right]\left(\theta-\overline{\theta}\right)\right]\left[U\left(q^{*}\right)-C\left(q^{*}\right)\right],
\end{aligned}$$

where the last inequality uses the fact that $\Delta \in [0, U(q^*) - C(q^*)]$ and the fact that $\theta > \alpha$ implies that $\beta\alpha(1-\theta) > \alpha - \theta[1-\beta(1-\alpha)]$. Thus, we have $\mu > \Theta$. Then by the previous proposition there exists $\mathcal{M}_E$ implementing the first best for the given μ as well. ■

Proposition 24 *Under competitive pricing,*

(a) If $\alpha = 1$ and $\theta < \underline{\theta}$, then the first-best allocation can be implemented by an e-money mechanism with limited transferability when $\mu \geq \bar{\mu}$. The first best allocation cannot be implemented by any e-money mechanism with limited participation.

(b) If $\theta \in (\alpha, \overline{\theta})$ and $(\beta+\alpha-1)U(q^*) < \alpha C(q^*)$, then the first-best allocation can be implemented by an e-money mechanism with limited participation when $\mu \geq \bar{\mu}$. The

first-best allocation cannot be implemented by any e-money mechanism with limited transferability.

Proof. Omitted here. ■

Proposition 25 *Given μ, under competitive pricing,*

(a) if there exists an e-money mechanism with limited transferability $\mathcal{M}_L = \{\Delta_b, \Delta_s, T_b(n), T_s(n)\}$ but not any e-money mechanism with limited participation $\mathcal{M}_E$, that implements the first best, then $\Delta > 0$ and $T_b(n_n) < 0$.

(b) if there exists $\mathcal{M}_E = \{T_b(n), T_s(n)\}$ but not any $\mathcal{M}_L$ that implements the first best, then $T_s(n) > 0$ and $T_b(n_n) < 0$.

Proof. Similar to the proof of Proposition 11. ■

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